1982

Power system transient stability using the critical energy of individual machines

Vijay Vittal
Iowa State University

INFORMATION TO USERS

This was produced from a copy of a document sent to us for microfilming. While the most advanced technological means to photograph and reproduce this document have been used, the quality is heavily dependent upon the quality of the material submitted.

The following explanation of techniques is provided to help you understand markings or notations which may appear on this reproduction.

1. The sign or "target" for pages apparently lacking from the document photographed is "Missing Page(s)". If it was possible to obtain the missing page(s) or section, they are spliced into the film along with adjacent pages. This may have necessitated cutting through an image and duplicating adjacent pages to assure you of complete continuity.

2. When an image on the film is obliterated with a round black mark it is an indication that the film inspector noticed either blurred copy because of movement during exposure, or duplicate copy. Unless we meant to delete copyrighted materials that should not have been filmed, you will find a good image of the page in the adjacent frame. If copyrighted materials were deleted you will find a target note listing the pages in the adjacent frame.

3. When a map, drawing or chart, etc., is part of the material being photographed the photographer has followed a definite method in "sectioning" the material. It is customary to begin filming at the upper left hand corner of a large sheet and to continue from left to right in equal sections with small overlaps. If necessary, sectioning is continued again—beginning below the first row and continuing on until complete.

4. For any illustrations that cannot be reproduced satisfactorily by xerography, photographic prints can be purchased at additional cost and tipped into your xerographic copy. Requests can be made to our Dissertations Customer Services Department.

5. Some pages in any document may have indistinct print. In all cases we have filmed the best available copy.

8221232

Vittal, Vijay

POWER SYSTEM TRANSIENT STABILITY USING THE CRITICAL ENERGY OF INDIVIDUAL MACHINES

Iowa State University PH.D. 1982

University Microfilms International 300 N. Zeeb Road, Ann Arbor, MI 48106

PLEASE NOTE:

In all cases this material has been filmed in the best possible way from the available copy. Problems encountered with this document have been identified here with a check mark __√__.

1. Glossy photographs or pages _____
2. Colored illustrations, paper or print _____
3. Photographs with dark background _____
4. Illustrations are poor copy _____
5. Pages with black marks, not original copy _____
6. Print shows through as there is text on both sides of page _____
7. Indistinct, broken or small print on several pages __√__
8. Print exceeds margin requirements _____
9. Tightly bound copy with print lost in spine _____
10. Computer printout pages with indistinct print _____
11. Page(s) __________ lacking when material received, and not available from school or author.
12. Page(s) __________ seem to be missing in numbering only as text follows.
13. Two pages numbered __________. Text follows.
14. Curling and wrinkled pages _____
15. Other ______________________________

Power system transient stability

using the critical energy

of individual machines

by

Vijay Vittal

A Dissertation Submitted to the

Graduate Faculty in Partial Fulfillment of the

Requirements for the Degree of

DOCTOR OF PHILOSOPHY

Major: Electrical Engineering

Approved:

Signature was redacted for privacy.

In Charge of Major Work

Signature was redacted for privacy.

For the Major Department

Signature was redacted for privacy.

For the Graduate College

Iowa State University
Ames, Iowa

1982

TABLE OF CONTENTS

TABLE OF CONTENTS

CHAPTER I. INTRODUCTION

Need for Direct Methods of Transient Stability Analysis

The term 'stability', when used by power systems engineers, is that property which ensures that the system will remain in operating equilibrium through the normal and abnormal operating conditions (1). As power systems grow larger and more complex, the stability studies gain paramount importance. With the ever increasing demand for electrical energy and dependence on an uninterrupted supply, the associated requirement of high reliability dictates that power systems be designed to maintain stability under specific disturbances, consistent with economy.

The issue of stability arises when the system is perturbed. The nature or magnitude of perturbation greatly affects the stability of the system. If the perturbation to which the power system is subjected is large, then the oscillatory transient will also be large. The question then becomes, whether after these swings, the system will settle to a new "acceptable" operating state, or whether the large swings will result in loss of synchronism. This is known as the transient stability problem. The large perturbation, which creates the transient stability problem, may be sudden change in load, a sudden change in reactance of the circuit caused, for instance, by a line outage or a "fault".

The conventional method to analyze transient stability is as follows: The transient behavior of the power system is described by a set of differential and algebraic equations; a time solution is obtained starting with the system condition prior to the initiation of the transient; and the time solution is carried out until it is judged that each of the synchronous machines maintains or loses synchronism with the rest of the system. It is to be noted that in a typical stability study the system is subjected to a sequence of disturbances. In obtaining the time solution, the appropriate equations describing the system and network parameters are used for each period in the study.

Transient stability studies are often conducted on power systems when they are subjected to faults. One of the important objectives of such studies is to ascertain whether the existing (or planned) switchgear and network arrangements are adequate for the system to withstand a prescribed set of disturbances without loss of synchronism being encountered. Alternatively the system planner, and the researcher, may seek answers in the form of the most severe fault, at a given location, that the system could withstand. In that case, the study yields the "critical clearing time" of faults at that location. This is useful for planning purposes, e.g., for comparing the relative robustness of network arrangements or for selection of switchgear.

The numerical method of stability analysis is very reliable, and has been widely used and accepted by the power industry. However, it has two major drawbacks.

1. The technique consists of numerically integrating a large number of differential equations for each fault case. A number of repeat simulations are thus required. Hence, in terms

of computational cost using the digital computer this method can be expensive.

2. There are certain situations in the day to day operation of a power system, where an operator would like to quickly estimate the degree of stability. These situations could arise due to certain unforseen circumstances, like equipment breakdown or line outage for maintenance purpose. Conventional stability analysis using repeat simulations are time consuming and hence may hinder the operator's decision.

It is because of these reasons that there is a definite need for direct methods of transient stability assessment. The direct methods in turn should satisfy the following requirements.

1. Predict transient stability (or instability) of a power system, when subjected to a given disturbance, reliably.

2. Provide a quick assessment of transient stability (in or near real time), to assist the system operator.

3. Perform the above functions at a reduced cost.

Review of Direct Methods

Early work on energy functions

Early work on the development of criteria for transient stability of power systems involved energy methods. These were "direct methods" in the sense that the transient stability was to be decided without obtaining a time solution of the machine rotor angles. The most familiar energy criterion for stability is the "equal area criterion". Reference (2) gives an excellent treatment of this topic. The equal area criterion simply states that the rotor of the perturbed synchronous machine will move until the kinetic energy acquired in motion during the faulted state, is totally converted into potential energy during the post-fault state. At this point, the acceleration of the rotor must be in the direction to reverse its motion. If these conditions are met, then the system damping is assumed to bring the machine to a new steady state operating point.

The energy criteria for a multi-machine power system have received more attention in the Soviet Union than in the West. In 1930, Gorev used the first integral of energy to obtain a criterion for stability(3). Magnusson (4) in 1947 developed a technique using the classical model with zero transfer conductances. His approach was very similar to that of Gorev's. In 1958, Aylett (5) published his work on "The energy-integral criterion of transient stability limits of power systems." He studied the phase-plane trajectories of a multi-machine system using the classical model. The most significant aspect of Aylett's work was the formulation of the system equations based on inter-machine movements. This provides a physical explanation to the dynamic behavior of the machines which determines whether synchronism is maintained. He also recognized that for an n-machine case the energy integral specifies a surface of degree $2(n-1)$. If this surface passes through a saddle point, it will, under certain conditions separate the regions of stable and unstable trajectories, thus forming a separatrix.

Work on Lyapunov's direct method

After the early work on energy methods, greater emphasis was given to shaping Lyapunov's direct method into an effective tool for the assessment of power system stability. Pioneering work in this area was done by Gless (6). The technique was applied to a single machine infinite bus example. El-Abiad and Nagappan (7) applied the method to a multi-machine system.

The basic concept in applying Lyapunov's direct method consists of writing the system differential equations in the post-fault state (after the final switching) in the form

$$\dot{\underline{x}} = \underline{f}(\underline{x}) \tag{1.1}$$

with the post-fault equilibrium state at the origin $\underline{0}$. A suitable Lyapunov function $V(\underline{x})$ is chosen, which along with its time-derivative $\dot{V}(\underline{x})$ has the required sign-definite properties. The stability region around the post-fault equilibrium state $\underline{0}$ is specified by an inequality of the form $V(\underline{x})<C$, where C is a constant to be determined. This constant is usually obtained by evaluating the function $V(\underline{x})$ at the boundary of the region of stability. This boundary was first (7) suggested to be a surface passing through the unstable equilibrium point (u.e.p.) closest to the post-fault stable equilibrium point (s.e.p.). To assess the transient stability of the power system, the Lyapunov method is applied to the post-fault power system. Thus, given conditions at the instant of fault clearing, the system V-function at that instant is computed. If $V<C$ system is stable, $V>C$ system is unstable. The critical clearing time is that instant at which $V=C$. Thus, explicit integration of the differential equation is done only during the fault period resulting in a marked reduction of computation time. The procedure appeared to have definite potential as a practical tool, but it required further refinement before it could be applied.

Efforts to make Lyapunov's method a feasible practial tool were directed on two fronts.

1. To obtain better Lyapunov functions (8-15). In order to construct valid Lyapunov functions, the transfer conductances were neglected (5,6,8,9) or represented using some approximations (15,16). This resulted in conservative estimates of the critical clearing time. In certain cases, as shown by Ribbens-Pavella (17), the exclusion of transfer conductances can be justified; however, it has been pointed out by Kitamura et al. (18) that for a heavily loaded system there exists a danger in

judging a practically unstable system to be stable if transfer conductance are neglected. In 1972 (19,20), a significant step forward was made using the inertial center transformation, as a result of which the energy contribuitng to the inertial center acceleration was subtracted since it did not contribute to instability.

2. To obtain better estimates of the region of stability (21-23), Prabhakara and El-Abiad (21) and Gupta and El-Abiad (22) obtained better regions of stability by choosing the u.e.p. based on fault location. Ribbens-Pavella et al. (23) provided a very interesting approach of first selecting an approximate u.e.p. and then improving upon the u.e.p. by determining the accelerating powers on the faulted trajectory.

The survey papers by Fouad (24) and Ribbens-Pavella (25) provide a very comprehensive review of the research conducted in applying Lyapunov's method to power systems.

Vector Lyapunov functions

Another technique applied to stability analysis by direct methods was that using vector Lyapunov functions. It was first proposed by Bellman (26) and Bailey (27). They demonstrated its usefulness for stability analysis of a complex composite system. Pai and Narayana (28) were the first to apply the technique to power systems, but the results obtained were very conservative. Using the work of Michel (29-31), Weissenberger (32), and Araki (33), Jocic, Ribbens-Pavella and Siljak (34) applied vector Lyapunov functions to power systems, but because of the majorization techniques involved the results obtained were very conservative. To date, Chen and Schinzinger (35), Pai and Vittal (36) have also applied vector Lyapunov functions to power systems with moderate success.

Recent work on energy functions

In 1979, System Control Incorporated (S.C.I.) (37, 38) published a report in which the overall objective was to develop the transient energy method into a practical tool for the transient stability analysis of power systems. The important accomplishments of the S.C.I. project were

1. Clear understanding and verification of the fact that by appropriately accounting for fault location in the transient energy method, the stability of a multi-machine system can be accurately assessed.

2. Development of techniques for the direct determination of critical clearing times. Approximate method of incorporating the effects of transfer conductances, accurate fault-on trajectory approximation and calculation of unstable equilibrium points.

3. Definition of the Potential Energy Boundary Surface (PEBS) which formed the basis for an important instability conjecture

and allowed for significant improvements in direct stability assessment.

The S.C.I. work had a few shortcomings. In certain complex modes of instability the correct u.e.p., could not be predicted. Also, when the critical trajectory did not pass close to an u.e.p., the results obtained were conservative. The technique using the PEBS gave conservative estimates of critical clearing time.

The concept of PEBS had been proposed by Kakimoto et al. (39) in 1978 using a Lure' type Lyapunov function. Bergen and Hill (40) developed a technique of constructing a Lyapunov function using the sparse network formulation, thus overcoming the problem of transfer conductances.

Fouad and co-workers (41-43) used a series of simulations on a practical power system to provide a physical insight into the instability phenomenon. Their conclusions can be summarized as follows.

1. Not all the excess kinetic energy at clearing contributes directly to the separation of the critical machines from the rest of the system. Some of that energy accounts for the other intermachine swings. For stability analysis, that component of kinetic energy should be subtracted from the energy that needs to be absorbed by the system for stability to be maintained.

2. If more than one generator tends to lose synchronism, instability is determined by the gross motion of these machines, i.e., by the motion of their center of inertia.

3. The concept of a controlling u.e.p. for a particular system trajectory is a valid concept.

4. First swing transient stability analysis can be made accurately and directly if:

   a. The transient energy is calculated at the end of the disturbance and corrected for the kinetic energy that does not contribute to system separation.

   b. The controlling u.e.p. and its energy are computed.

Finally, the most recent work in the area of energy functions has been done by Athay and Sun (44). Using a nonlinear load model, they developed a new topological energy function.

Motivation for present work

The research efforts reviewed approached power system stability from a system-wide viewpoint. It has been a common practice to develop an energy function or a Lyapunov function for the entire system. Transient stability in a power system is a very interesting phenomenon. Instability usually occurs in the form of one machine or a group of machines losing synchronism with respect to the other machines in the system. In other words, instability occurs according to the behavior of this particular group of machines. Hence, the prediction of stability using an energy function representing the total system energy may result in some conservativeness. Another point to be noted is that the stability analysis using system-wide energy function does not give any indication of the mode of instability i.e., one cannot always predict the machine or the group of machines losing synchronism. Furthermore, the use of a system-wide energy function may mask the mechanism by which the transient energy is exchanged among the machines, and where the energy resides in the network. Hence it is often not clear how loss of synchronism takes place. Thus, the concept of examining system stability by a system-wide function becomes suspect, and the need for generating energy functions for individual machines (or for groups of machines) becomes obvious. This research work develops such functions and examines their use for transient stability analysis of a multi-machine power system.

Scope of the Work

The objectives of this research endeavor are:

1. Develop an expression for the energy function of an individual machine or for a group of machines.

2. Explain the mechanics of stability (or instability) for a multi-machine power system by accounting for energy of individual machines or group of machines.

3. Develop a technique to predict the mode of instability using the individual machine energy function.

4. Provide a comparison with the system wide energy function and arrive at a correspondence between the critical energy of individual machines and the total system critical energy at the controlling u.e.p.

5. Conduct simulation and validation studies on practical power systems. Throughout the course of analysis, two power networks were used. A 17-generator system, representative of the power network of the State of Iowa, and a 20-generator IEEE test system.

CHAPTER II. TRANSIENT ENERGY OF INDIVIDUAL MACHINES

Individual Machine Transient Energy

As explained in the previous chapter, the main aim of this work is to explain the phenomenon of "first swing" transient stability (or instability) using the energy of individual machines or groups of machines. In this investigation, the simplest model representing a multi-machine power system is used. In the literature, it is commonly known as the classical model (see Chapter 2 of (45)). A number of simplifying assumptions are made in arriving at the classical model. These are:

1. The transmission network is modeled by steady state equations.
2. Mechanical power input to each generator is constant.
3. Damping or asynchronous power is negligible.
4. The synchronous machine is modeled by constant voltage behind transient reactance.
5. The motion of the rotor of a machine coincides with the angle of the voltage behind the transient reactance.
6. Loads are represented by constant passive impedances.

For the classical model being considered, the equations of motion are:

$$M_i\dot{\omega}_i = P_i - P_{ei}$$

$$i = 1, 2, ---, n \qquad (2.1)$$

$$\dot{\delta}_i = \omega_i$$

where

$$P_{ei} = \sum_{\substack{j=1 \\ j\neq i}}^{n} [C_{ij}\sin(\delta_i-\delta_j) + D_{ij}\cos(\delta_i-\delta_j)] \qquad (2.2)$$

$$P_i = P_{mi} - E_i^2G_{ii}$$

$$C_{ij} = E_iE_jB_{ij},\ D_{ij} = E_iE_jG_{ij}$$

P_{mi} - mechanical power input

G_{ii} - driving point conductance

E_i - constant voltage behind the direct axis transient reactance.

ω_i; δ_i - generator rotor speed and angle deviations, respectively with respect to a synchronously rotating reference frame.

M_i - moment of inertia

$B_{ij}(G_{ij})$ - transfer susceptance (conductance) in the reduced bus admittance matrix.

Equations (2.1) are written with respect to an arbitrary synchronous reference frame. Transformation of these equations to the inertial center coordinates not only offers physical insight into the transient stability problem formulation in general, but also removes the energy associated with the inertial center acceleration which does not contribute to the stability determination (19,20). Referring to equations (2.1) define

$$M_T = \sum_{i=1}^{n} M_i$$

$$\delta_o = \frac{1}{M_T} \sum_{i=1}^{n} M_i \delta_i$$

then

$$M_T \dot{\omega}_o = \sum_{i=1}^{n} P_i - P_{ei} = \sum_{i=1}^{n} P_i - 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} D_{ij} \cos\delta_{ij} = P_{COI}$$

$$\dot{\delta}_o = \omega_o \qquad (2.3)$$

The generators' angles and speeds with respect to the inertial center are given by

$$\theta_i = \delta_i - \delta_o$$

$$i = 1, 2, ---, n \qquad (2.4)$$

$$\tilde{\omega}_i = \omega_i - \omega_o$$

and in this coordinate system the equations of motion become

$$M_i \dot{\tilde{\omega}}_i = P_i - P_{ei} - \frac{M_i}{M_T} P_{COI}$$

$$i = 1, 2, ---, n \qquad (2.5)$$

$$\dot{\theta}_i = \tilde{\omega}_i$$

The transient energy of each machine can be derived directly from the swing equations written with respect to the inertial center following the steps outlined below.

- Multiply the i^{th} post-fault swing equation (2.5) by $\dot{\theta}_i$ and rearrange the terms to get

$$[\,M_i \dot{\tilde{\omega}}_i - P_i + P_{ei} + \frac{M_i}{M_T} P_{COI}\,]\,\dot{\theta}_i \qquad i = 1, 2, ---, n \tag{2.6}$$

- Integrate (2.6) with respect to time using as a lower limit $t = t_s$ where $\underline{\tilde{\omega}}(t_s) = 0$ and $\underline{\theta}(t_s) = \theta^s$ is the stable equilibrium point (s.e.p.), yielding

$$V_i = \frac{1}{2} M_i \tilde{\omega}_i^2 - P_i(\theta_i - \theta_i^s) + \sum_{\substack{j=1 \\ j \neq i}}^{n} C_{ij} \int_{\theta_i^s}^{\theta_i} \sin\theta_{ij}\, d\theta_i + \sum_{\substack{j=1 \\ j \neq i}}^{n} D_{ij} \int_{\theta_i^s}^{\theta_i} \cos\theta_{ij}\, d\theta_i + \frac{M_i}{M_T} \int_{\theta_i^s}^{\theta_i} P_{COI}\, d\theta_i \tag{2.7}$$

$$i = 1, 2, ---, n$$

Interpretation of V_i

A physical interpretation of V_i is provided as follows. The expression for V_i when closely examined contains terms representing the change in kinetic energy and potential energy due to the motion of the rotor of machine i. The former is easily identified as the first term in equations (2.7). The change in potential energy (PE) includes three main components.

1. Change in PE due to the power flow out of node i. Consider a single branch between nodes i and j going from equilibrium position θ_{ij}^s to some transient angle θ_{ij}. The electric power flow in this branch, from i to j and from j to i, is given by

$$P_{ij} = C_{ij}\sin\theta_{ij} + D_{ij}\cos\theta_{ij}$$
$$P_{ji} = C_{ji}\sin\theta_{ji} + D_{ji}\cos\theta_{ji} \tag{2.8}$$

The branch ij will have a change in potential ΔPE due to electrical power flow given by

$$\Delta PE = \int_{\theta_i^s}^{\theta_i} P_{ij}d\theta_i + \int_{\theta_j^s}^{\theta_j} P_{ji}d\theta_j \tag{2.9}$$

The first term in (2.9) is associated with node i, while the second term is associated with node j. Since the network has been reduced to the internal generator nodes, each node will have (n-1) branches connecting it to all the other (n-1) nodes. Each one of these branches will have a contribution to the change in potential energy associated with the power flow out of the node, similar to one of the terms of (2.9). This portion of potential energy change is identified as the third and fourth terms in the expression for V_i given by (2.7).

2. Change in potential energy due to the change in rotor position between θ_i^s and θ_i. This change is given by the second term in (2.7).

3. Change in potential energy due to the i^{th} machine contribution to the acceleration of the center of inertia (COI). This change, given by the last term in (2.7), arises from the

portion of the power flow out of machine i contributing to the motion of the COI. Equations (2.7) can hence be written in a concise form as

$$V_i = V_{KEi} + V_{PEi} \qquad i = 1, 2, ---, n \qquad (2.10)$$

Since the machine nodes are retained intact and not approximated by an equivalent, the transient energy function thus obtained gives the correct expression for the energy interchange between machine i and every other machine in the system.

Comparison with equal area criterion

The correspondence of the equal area criterion and the transient energy method for a two-machine system is illustrated for the equivalent single machine infinite bus system in Figure 2.1. In this figure, two plots with the same abscissa are shown. The upper plot illustrates the familiar equal area criterion in which the critical clearing angle is defined by the equality of areas A_1 and A_2. The lower energy plots illustrate the transient energy method which can be used to specify the critical angle in terms of potential and kinetic energy as shown. $PE(\delta^u)$ is the maximum value of potential energy and occurs at the angle δ^u. It provides a measure of the energy absorbing capacity of the system, and is called the critical energy. In the transient energy method, the excess kinetic energy contributing to instability during the fault-on period is added to the potential energy at the corresponding angle coordinate. This gives the total energy at clearing. The total

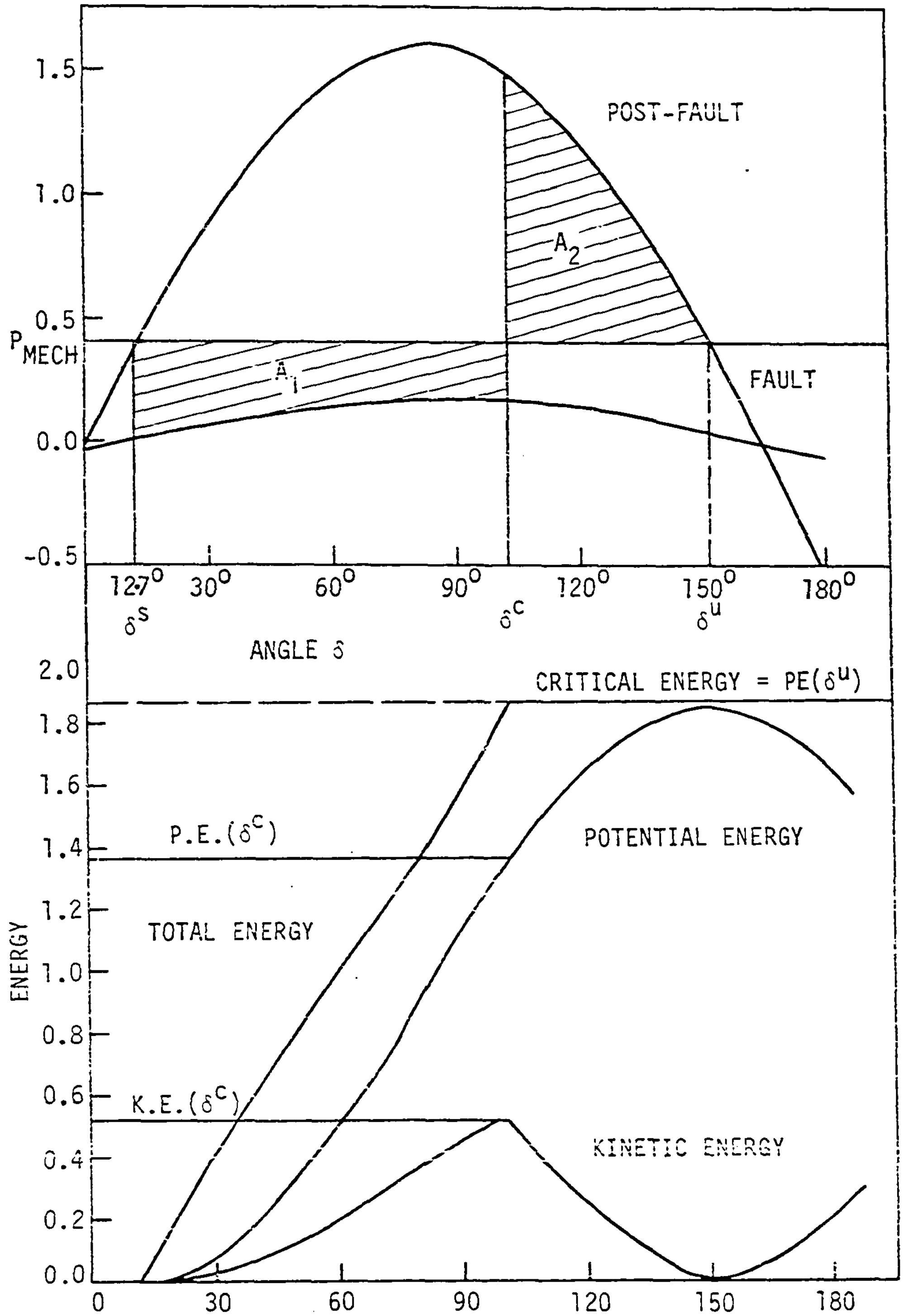


Figure 2.1. Comparison of equal area criteria and transient energy method for a two machine system

energy at clearing is compared with the value of critical energy. The system becomes unstable when the total energy exceeds the critical energy. The critical clearing angle is defined when the total energy at clearing just becomes equal to the critical energy.

For a system with three or more machines, the direct analysis becomes more difficult. The critical energy is not defined, and the crux of the problem lies in its determination. It is in this step that the proposed approach of accounting for individual machine energy differs significantly from those adopted previously, based on system-wide energy. In effect, the energy for each individual machine is similar to the equal area criterion, where instead of the infinite bus equivalent, the rest of the system is modeled accurately, thus preserving the structure of the system.

Transient Energy of a Group of Machines

Without loss of generality, the procedure is illustrated using a two machine group.

Potential energy of the group

Consider an n-machine system with machines 1 and 2 forming the critical group. The potential energy of these two machines with respect to the rest of the system is given by

$$V_{PE1,2} = V_{PE1} + V_{PE2} - (\Delta PE \text{ due to power flow in branches between nodes 1 and 2}) \qquad (2.11)$$

substituting the appropriate terms in (2.11) yields.

$$V_{PE1,2} = -\sum_{i=1}^{2} P_i(\theta_i-\theta_i^s) + \sum_{i=1}^{2}\sum_{j=3}^{n} C_{ij}\int_{\theta_i^s}^{\theta_i} \sin\theta_{ij}d\theta_i$$
$$+ \sum_{i=1}^{2}\sum_{j=3}^{n} D_{ij}\int_{\theta_i^s}^{\theta_i} \cos\theta_{ij}d\theta_i + \frac{1}{M_T}\sum_{i=1}^{2} M_i\int_{\theta_i^s}^{\theta_i} P_{COI}d\theta_i \qquad (2.12)$$

In general if machines, 1, 2, ---, k form the critical group and machines k+1, ---, n the rest of the system then

$$V_{PE1,2,---,k} = -\sum_{i=1}^{k} P_i(\theta_i-\theta^s{}_i) + \sum_{i=1}^{k}\sum_{j=k+1}^{n} [\,C_{ij}\int_{\theta_i^s}^{\theta_i} \sin\theta_{ij}d\theta_i$$
$$+ D_{ij}\int_{\theta_i^s}^{\theta_i} \cos\theta_{ij}d\theta_i\,] + \frac{1}{M_T}\sum_{i=1}^{k} M_i\int_{\theta_i^s}^{\theta_i} P_{COI}d\theta_i \qquad (2.13)$$

Equivalent Kinetic Energy of the Group

It was shown in references (41,43) that the transient kinetic energy responsible for the separation of the critical generators from the rest of the system is that of the motion between the center of inertia of the critical group and the center of inertia of the rest of the machines. By designating the moment of inertia and the speed of the critical machines as M_{cr} and $\tilde{\omega}_{cr}$, and the corresponding quantities for the remaining machines as M_{sys} and $\tilde{\omega}_{sys}$, the kinetic energy of the equivalent two machine groups is given by

$$V_{KE} = \frac{1}{2} M_{eq}\, \tilde{\omega}_{eq}^2 \qquad (2.14)$$

where

$$M_{eq} = M_{cr} M_{sys}/(M_{cr} + M_{sys})$$

$$\tilde{\omega}_{eq} = (\tilde{\omega}_{cr} - \tilde{\omega}_{sys}) \qquad (2.15)$$

The total transient energy of the group is readily obtained from (2.12), (2.13)

$$V_{1,2} = V_{PE1,2} + V_{KE1,2} \qquad (2.16)$$

Relation with System Wide Energy

In reference (38), a system wide energy function was derived from the swing equations in the inertial center coordinates, in the following manner

- Multiply the i^{th} post-fault swing equation by $\dot{\theta}_i$ and form the sum

$$\sum_{i=1}^{n} [M_i \dot{\tilde{\omega}}_i - P_i + P_{ei} + \frac{M_i}{M_T} P_{COI}] \dot{\theta}_i$$
$$= \sum_{i=1}^{n} F_i (\dot{\tilde{\omega}}_i, \underline{\theta}) \dot{\theta}_i \qquad (2.17)$$

- Using equalities $C_{ij} = C_{ji}$ and $D_{ij} = D_{ji}$ integrate (2.17) with respect to time to obtain

$$V = \frac{1}{2}\sum_{i=1}^{n} M_i\tilde{\omega}_i^2 - \sum_{i=1}^{n} P_i(\theta_i - \theta_i^s)$$

$$- \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} C_{ij}(\cos\theta_{ij} - \cos\theta_{ij}^s) \tag{2.18}$$

$$+ \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} D_{ij}\int_{\theta_i^s+\theta_j^s}^{\theta_i+\theta_j} \cos\theta_{ij}\, d(\theta_i + \theta_j)$$

Also,

$$V = \int\left[\sum_{i=1}^{n} F_i(\dot{\tilde{\omega}}_i,\underline{\theta})\dot{\theta}_i\right] dt \tag{2.19}$$

Since the summation is over a finite range,

$$V = \sum_{i=1}^{n} \int F_i(\dot{\tilde{\omega}}_i, \underline{\theta})\, \dot{\theta}_i\, dt \tag{2.20}$$

The right hand side of (2.20) when evaluated results in the expression obtained in (2.7). Hence,

$$V = \sum_{i=1}^{n} V_i \tag{2.21}$$

Thus, the sum of the individual machine energies is equal to the total system energy.

CHAPTER III. CRITICAL ENERGY OF INDIVIDUAL MACHINES

Critical Energy

From the previous chapter the transient energy of each individual machine from Equation (2.7) is repeated for convenience.

$$V_i = \frac{1}{2} M_i \tilde{\omega}_i^2 - P_i(\theta_i - \theta_i^s) + \sum_{\substack{j=1 \\ j\neq i}}^{n} C_{ij} \int_{\theta_i^s}^{\theta_i} \sin\theta_{ij} d\theta_i + \sum_{\substack{j=1 \\ j\neq i}}^{n} D_{ij} \int_{\theta_i^s}^{\theta_i} \cos\theta_{ij} d\theta_i + \frac{M_i}{M_T} \int_{\theta_i^s}^{\theta_i} P_{COI}\, d\theta_i \qquad i = 1, 2, ---, n \tag{3.1}$$

Examining equation (3.1) it can be seen that the transient energy of machine i depends on the post disturbance network and the position and speed of machine i in relation to other machines. Thus, the components of transient energy of machine i vary along the post disturbance trajectory. However, as that machine pulls away from the rest of the system, its kinetic energy is being converted into potential energy. Therefore, that machine will continue to move away from the system until the kinetic energy which it possessed at the instant the disturbance was removed is totally absorbed by the network, i.e., converted to potential energy. When this takes place, the machine will move toward the rest of the system and stability is maintained. If the kinetic energy is not totally absorbed by the network, the machine will continue to move away from the other machines, losing synchronism in the process.

Intuitively, it can be noted that the network's ability to absorb the kinetic energy of machine i and convert it into potential energy is the key factor in determining whether, for a given disturbance, machine i will maintain synchronism with the rest of the system. Since this potential energy varies along the post disturbance trajectory, a number of questions immediately present themselves: does it have a maximum value along the trajectory? Does this maximum value (if it exists) vary with different disturbances resulting in different modes of instability? How can this value be determined? These questions are inter-related; they represent the central issues dealt with, in the course of this investigation.

Numerous simulations have been conducted on three systems. It was found that if the fault is kept long enough for one or more machines to become critically unstable, the potential energy of the critical machine goes through a maximum before instability occurs. Furthermore, this maximum value (of the potential energy along the post disturbance trajectory) of a given machine, has been found to be essentially independent of the duration of the disturbance and mode of instability. This value of potential energy for machine i is the critical value of V_i, i.e.,

$$V_{i/\text{critical}} = V_{PEi/\text{max along trajectory}} \tag{3.2}$$

Flatness of $V_{i/critical}$

A heuristic justification of tha assumption is provided by examining the effect of a fault on a power system. If the fault is cleared soon enough, all the machines will remain in synchronism and the system will be stable. If the duration of the fault is extended to an instant just beyond the critical clearing time, the system will barely become unstable, i.e., one or more machines will lose synchronism. These machines are known as the critical group. For the critical group, the network's energy absorbing capacity (ability to convert to potential energy) is not sufficient to convert all the kinetic energy it possessed at the instant the disturbance was removed. In other words, the value of energy at clearing exceeds $V_{critical}$ for this group along its trajectory.

If the fault remains longer than the instant of critically unstable condition discussed above, more transient energy is injected into the system by the disturbance. This fault energy distributes itself among the machines, including the critical group that had become unstable in the critically unstable condition. Additional energy injected into those machines will not alter their situation since they had already lost synchronism. Thus, the value of $V_{critical}$ for this group will not be altered by the increase of the severity of the disturbance.

To obtain a better insight into the argument, consider a situation in which two machines i and j are severely disturbed by the fault. In the critically unstable case however, only machine i loses synchronism, while machine j is stable. Assume that when the fault duration is extended both machines become unstable, changing the mode of instability from i alone to i and j losing synchronism. From the above discussion, it can be noted that the critical energy for machine i is not affected. For machine j, the terms making up the potential energy in Equation (3.1) will hardly be affected. Their magnitudes will be determined by the rotor position of machine j with respect to the rest of the machines. Minor adjustments in their magnitude may occur, but the potential energy maximum for machine j will remain essentially constant.

The important point being made here is that whether the fault is barely sufficient to drive the critical machines to instability, or whether it is sustained so that additional machines may lose synchronism, it is the critical machines that go unstable first. In either case, the critical machines "see" essentially the same system, i.e., the same group of machines that remain stable. For this group, the network's ability to absorb its transient energy and convert it to potential energy is a fixed quantity. Instability will occur only if its initial energy exceeds this limit.

Evaluation of Critical Energy

Determination of the mode of instability

It has been established in the previous section that, for a given type of disturbance, i.e., disturbance location and post disturbance network, the value of $V_{critical}$ for a given machine is essentially independent of the duration of the disturbance. Therefore, a convenient method to determine the mode of instability is provided by examining the sustained fault trajectory. Also, the sustained fault constitutes the most severe disturbance for a given fault location. Hence, the values of $V_{critical}$ obtained on the sustained fault trajectory will always provide a safe estimate of the individual machine critical energy.

By simulating a sustained fault (or a fault of long duration), the potential energy term of equation (3.1) are computed V_{PEi}, i = 1, 2, ---, n for each instant of time. The values of V_{PEmax} are noted for the different machines (or groups of machines). These represent the value of $V_{critical}$ i.e.,

$$V_{critical/i} = V_{PEmax/i} \tag{3.2}$$

The value of V_{PEmax} obtained represent the energy absorbing capacity for each machine. It gives a measure of the amount of kinetic energy converted to potential energy.

To determine whether instability occurs, the total transient energy at the instant of fault clearing is compared with the value of $V_{critical}$ for each machine. The mode of instability is then given by those machines whose transient energy at clearing exceeds their critical energy

Calculation of $V_{critical}$ for individual machines

Examining equation (3.1) it can be seen that the integrands are not independent of the trajectory. Therefore, the individual machine energies along the faulted trajectory are evaluated numerically using the trapezoidal rule. A special computer program was developed to obtain the individual machine energies at each instant. This program has been used in the simulations presented later on in this research endeavor. Details of this program are given in the Appendix.

CHAPTER IV. TEST NETWORKS FOR VALIDATION

The Three Test Systems

This research used the individual machine energy function to analyze several faults on three test systems. They are a 4-generator system, a 17-generator equivalent of the power network of the 'State of Iowa' and a 20-generator IEEE test system.

The 4-generator test system

This test system, shown in Figure 4.1, is a modified version of the 9-bus, 3-machine, 3-load system widely used in the literature and often referred to as the WSCC test system. The modifications adopted are:

- Changing the rating of the transmission network from 230kV to 161kV to avoid having an excess VAR problem; the R and X values of the lines in per unit remain the same.

- Adding a fourth generator, connected to the original network by a step-up transformer and a double-circuit, 120-mile, 161-kV transmission line; the new generator has the same rating as one of the original generators. The new system has a generation capacity of 680MW.

The generator data and the initial operating condition are given in Table 4.1. This small test system was used primarily for validation

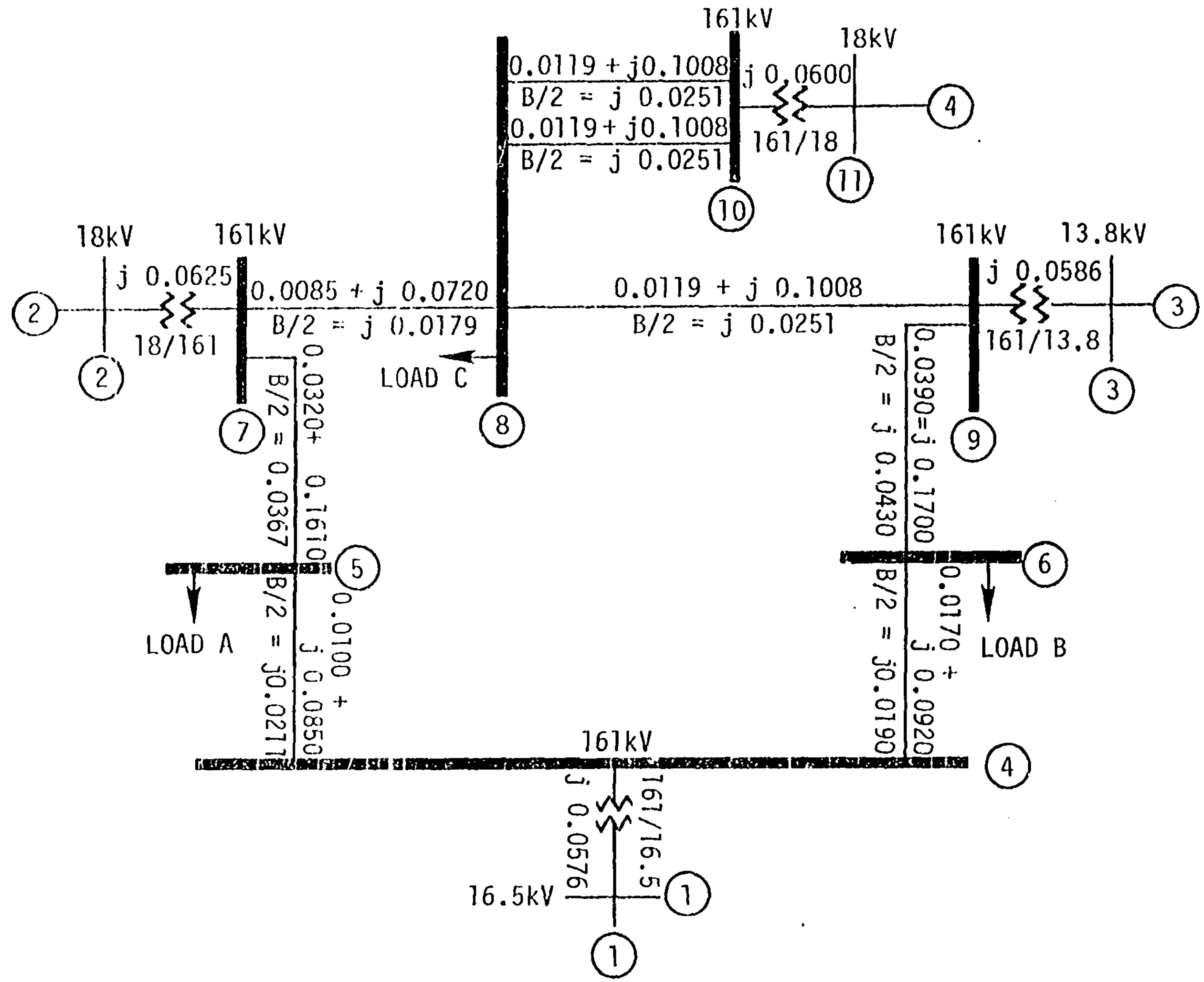


Figure 4.1. 4-generator test power system

Table 4.1. Generator data and initial conditions

Generator Number	Generator Parameters[a] H (MW/MVA)	Generator Parameters[a] x'_d (pu)	Initial Conditions P_{mo}[a] (pu)	Initial Conditions: Internal Voltage E (pu)	Initial Conditions: Internal Voltage δ (degrees)
4-Generator System					
1	23.64	0.0608	2.269	1.0967	6.95
2	6.40	0.1198	1.600	1.1019	13.49
3	3.01	0.1813	1.000	1.1125	8.21
4	6.40	0.1198	1.600	1.0741	24.90
17-Generator System					
1	100.00	0.004	20.000	1.0032	-27.92
2	34.56	0.0437	7.940	1.1333	-1.37
3	80.00	0.0100	15.000	1.0301	-16.28
4	80.00	0.0050	15.000	1.0008	-26.09
5	16.79	0.0507	4.470	1.0678	-6.24
6	32.49	0.0206	10.550	1.0505	-4.56
7	6.65	0.1131	1.309	1.0163	-23.02
8	2.66	0.3115	0.820	1.1235	-26.95
9	29.60	0.0535	5.517	1.1195	-12.41
10	5.00	0.1770	1.310	1.0652	-11.12
11	11.31	0.1049	1.730	1.0777	-24.30
12	19.79	0.0297	6.200	1.0609	-10.10
13	200.00	0.0020	25.709	1.0103	-38.10
14	200.00	0.0020	23.875	1.0206	-26.76
15	100.00	0.0040	24.670	1.0182	-21.09
16	28.60	0.0559	4.550	1.1243	-6.70
17	20.66	0.0544	5.750	1.116	-4.35

[a]On 100-MVA base.

of new procedures and/or computer programs developed. For faults at or near Generator No. 4, the mode of instability is simple and the system's dynamic behaviour is predictable.

The 17-generator test system

The Power System Computer Service of Iowa State University has been involved in several full-scale stability studies for new generating units in the Iowa area. The Philadelphia Electric Transient Stability Program was used in these studies. The base set of data and the results of one of these studies, the NEAL 4 stability study, were used to develop a Reduced Iowa System model, shown in Figure 4.2.

The generator data and initial operating conditions are given in Table 4.1. Load flow data are given in reference (43)[1].

This test system was used to simulate faults primarily in the western part of the network along the Missouri river. Several generating plants are located in this area. A disturbance in that part of the network substantially influences the motion of several generators. Thus, very complex modes of instability can occur, offering a severe test to the technique developed.

---

[1]The 17-generator equivalent of the Iowa Network was developed and tested by Dr. K. Kruempel, Iowa State University, Ames, Iowa, in the research project reported upon in reference (43).

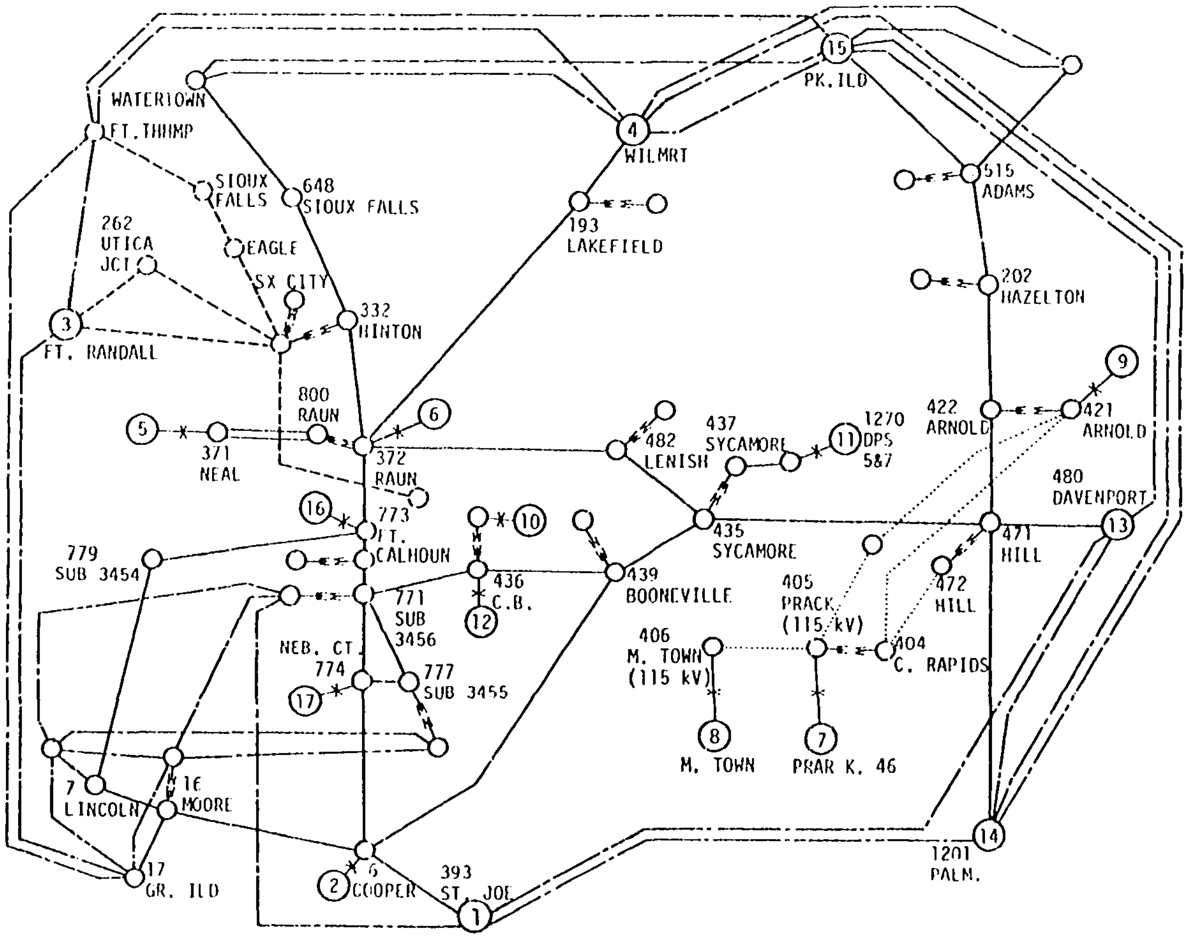


Figure 4.2. 17-generator system (Reduced Iowa System)

The 20-generator test system

This system is shown in Figure 4.3. It is known as the IEEE test system and has 118-buses. This system was investigated in reference (46).

The generator data and initial operating conditions are given in Table 4.2. Load flow data are given in reference (38).

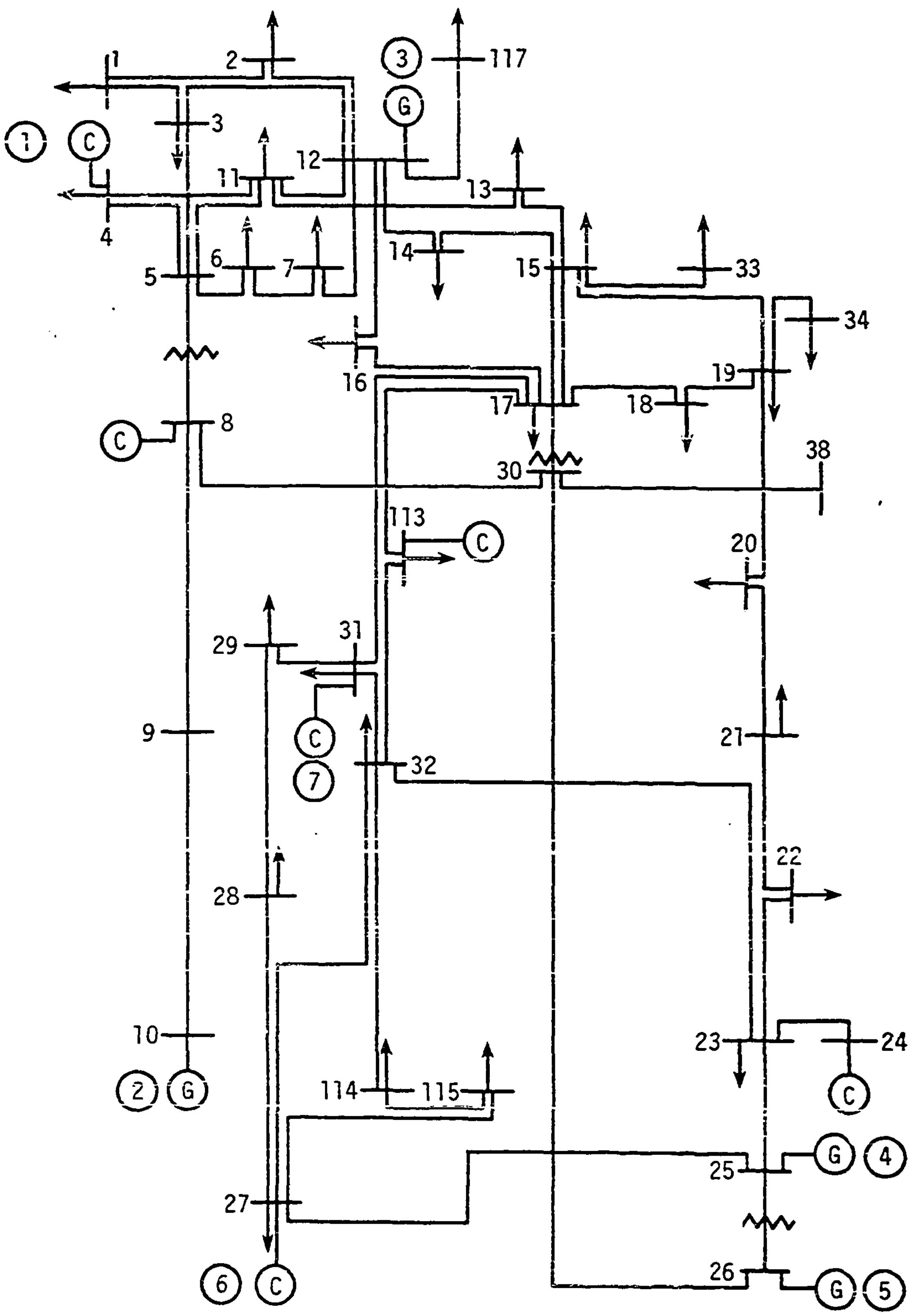
This test system was used to simulate three-phase faults at seven different locations near the terminal of generators or synchronous condensers. In all cases, the fault is cleared without line switching (to compare results with those previously published (38)). In Figure 4.3, the machine numbers are indicated within the circle.

Table 4.2. Generator data and initial conditions

Generator Number	Generator Parameters[a] H (MW/MVA)	Generator Parameters[a] x'_d (pu)	Initial Conditions P_{mo}[a] (pu)	Initial Conditions: Internal Voltage E (pu)	Initial Conditions: Internal Voltage δ (degrees)
20-Generator System					
1	8.00	0.0875	-0.0900	0.9875	-14.885
2	22.00	0.0636	4.5000	1.0941	20.443
3	8.00	0.1675	0.8500	1.1801	-10.216
4	14.00	0.1000	2.2000	1.1269	8.944
5	26.00	0.0538	3.1400	1.0516	9.121
6	8.00	0.0875	-0.0900	0.9778	-14.868
7	8.00	0.0875	0.0700	1.0005	-16.644
8	8.00	0.0875	-0.4600	1.0027	-24.929
9	8.00	0.0875	-0.5900	1.0286	-24.282
10	12.00	0.1166	2.0400	1.2061	2.131
11	10.00	0.1591	1.5500	1.1340	0.653
12	12.00	0.1166	1.6000	0.9782	5.185
13	20.00	0.0700	3.9100	1.1478	11.448
14	20.00	0.0700	3.9200	1.0837	11.516
15	30.00	0.0466	5.1430	1.0329	12.972
16	28.00	0.0500	4.7700	1.1253	10.720
17	32.00	0.04375	6.0700	1.0409	24.265
18	8.00	0.0875	-0.8500	1.0429	-0.974
19	16.00	0.0875	2.5200	1.1500	8.869
20	15.00	0.0466	-0.4300	0.9958	-16.226

[a]On 100-MVA base.

Figure 4.3a. IEEE test system part I


1
2
3
117
G
C
4
5
6
7
11
12
13
14
15
33
34
16
17
18
19
8
C
30
38
113
C
20
29
31
C
7
9
32
21
22
28
10
2
G
114
115
23
24
C
25
G
4
27
26
G
5
6
C

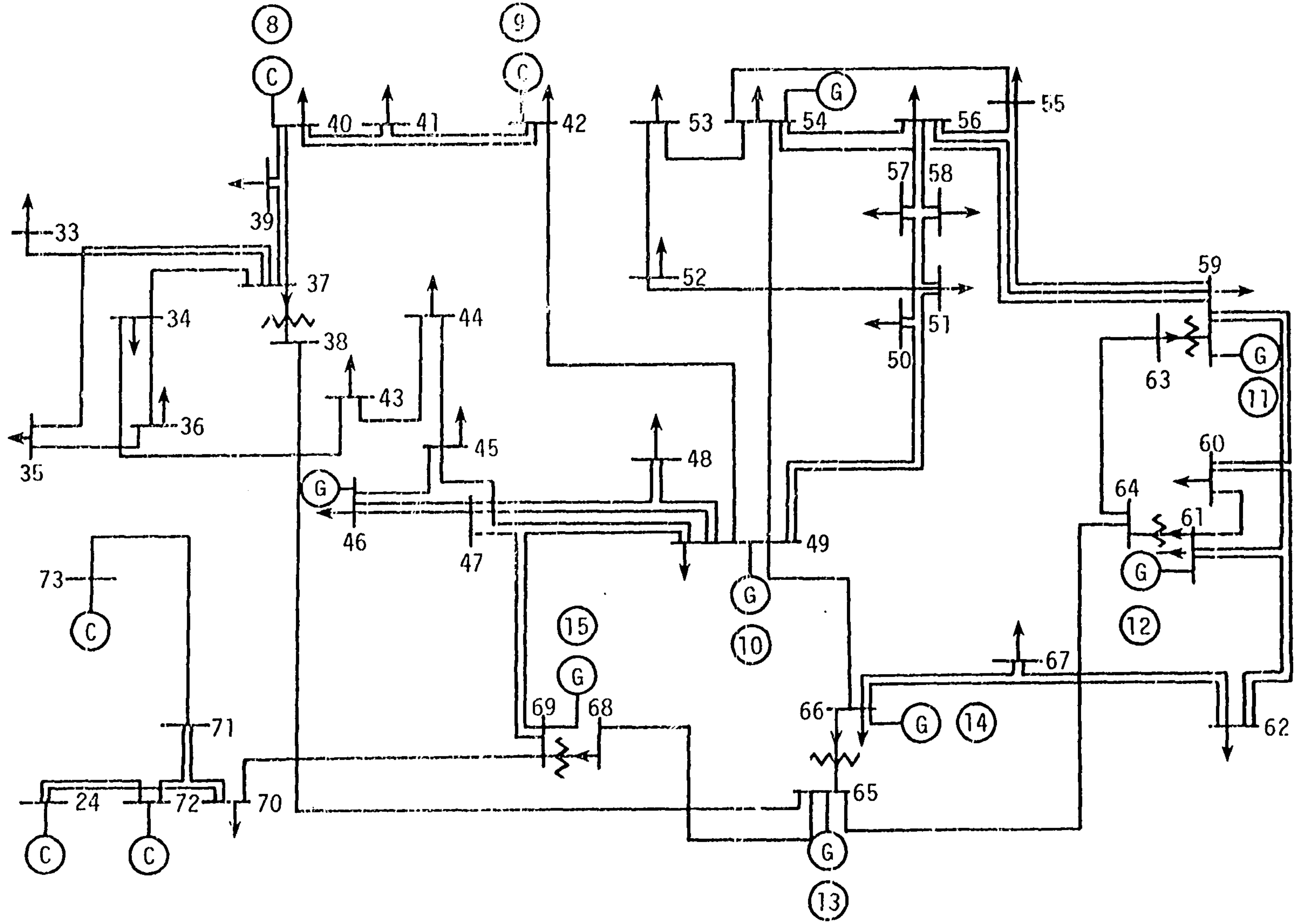


Figure 4.3b. IEEE test system part II

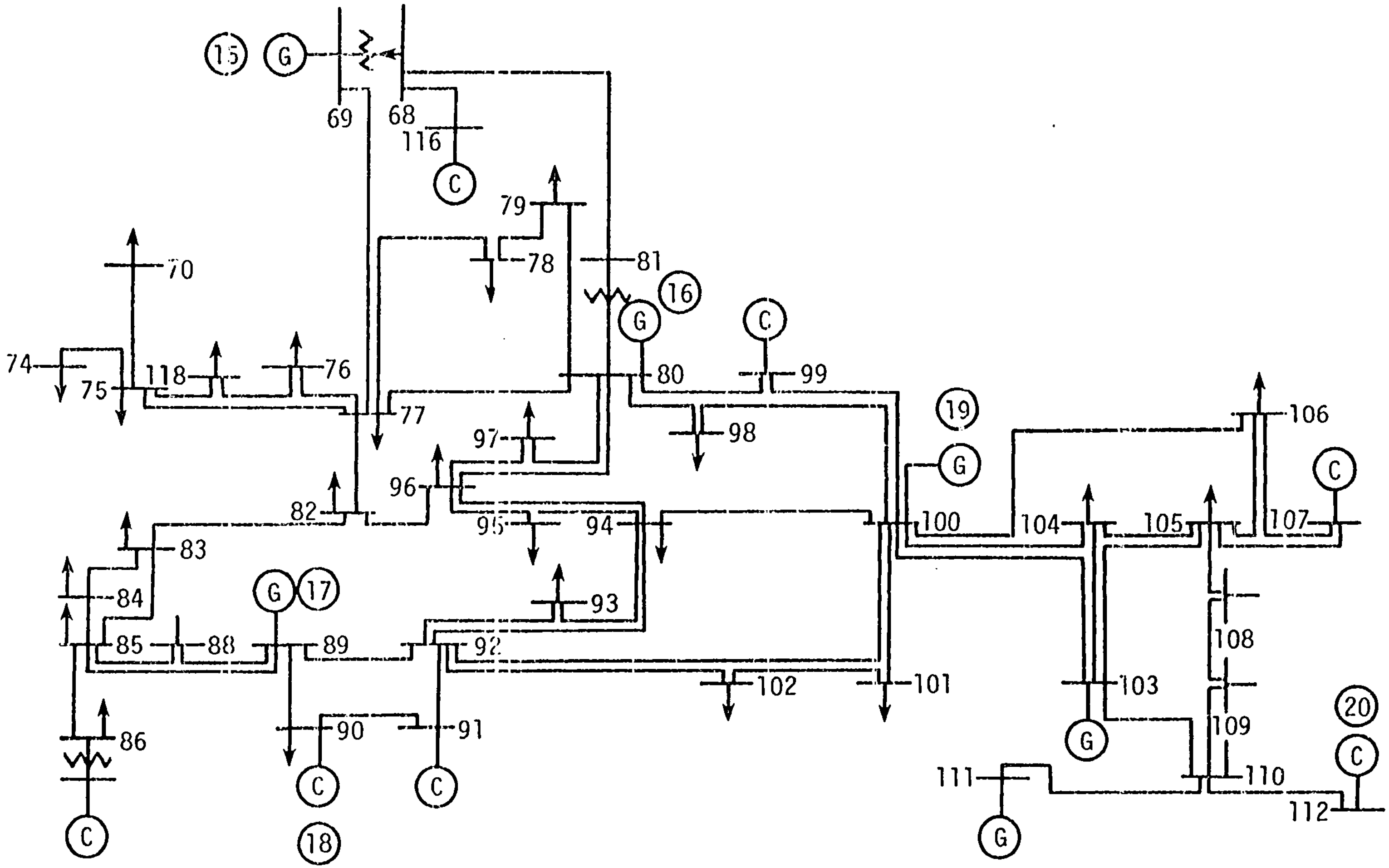


Figure 4.3c. IEEE test system part III

CHAPTER V. TRANSIENT STABILITY ASSESSMENT USING THE INDIVIDUAL MACHINE ENERGY

Procedure for Transient Stability Assessment

The procedure for transient stability assessment using the transient energy of individual machines (or groups of machines) is outlined below.

Step 1: For the post-disturbance network, the stable equilibrium point $\underline{\theta}^s$ and the reduced short circuit admittance matrix Y_{BUS} are determined.

Step 2: For each of the candidate fault locations, a sustained fault case is run. Typically for a period of 1 second or less. The values of V_i, $i = 1, 2, ---, n$, are computed along the faulted trajectory using the special computer program. (See the Appendix.)

Step 3: By examining V_i and its components, the values of $V_{i/critical} = V_{PEi/max}$ are determined and stored for each fault location.

Step 4: For a given disturbance, the values of θ_i and $\tilde{\omega}_i$ are obtained at the end of the disturbance, e.g., at fault clearing. From this information, $V_{i/t = t_c}$ is computed. A correction is made for the kinetic energy term as in equation (2.14).

Step 5: Transient stability is checked. Machine i will be stable or unstable depending on $V_{i/t=t_c} \lessgtr V_{i/critical}$. For a group of more than one machine going unstable, the above criterion holds for each machine in the group. In addition, the value of V_i for the group (as given by equation (2.16)) must exceed its critical value.

Results

The 4-generator test system

The fault investigated is a three-phase fault at Bus 10 cleared by opening one of the lines 8-10.

The 17-generator test system

The following faults were investigated

- A three-phase fault at Raun(Bus No. 372), cleared by opening line 372-193.
- A three-phase fault at Council Bluffs (C.B.) unit no. 3 (Bus No. 436), cleared by opening line 436-771.
- A three-phase fault at Ft. Calhoun (Bus No. 773), cleared by opening line 773-339.
- A three-phase fault at Cooper (Bus No. 6), cleared by opening line 6-774.

The 20-generator test system

Three-phase faults were applied at generator or synchronous condenser terminals and cleared without line switching. The fault cases were.

- Fault at terminal of generator #2
- Fault at terminal of generator #3
- Fault at terminal of generator #4
- Fault at terminal of generator #5
- Fault at terminal of generator #9
- Fault at terminal of generator #13
- Fault at terminal of generator #18

Critical Transient Energy

For the faults investigated, the maximum potential energy of the critical machines, i.e., the machines that first become unstable, is computed for the critically unstable condition and for the sustained fault case. This information is displayed in Table 5.1. The data in Table 5.1 clearly show that the maximum potential energy (along the faulted trajectory) for the critical machines is fairly constant for a variety of modes of instability for the same disturbance location. For example, the sustained faults at C.B. #3, Ft. Calhoun, and Cooper in

Table 5.1. Critical transient energy for critical machines

Fault Location	Critically Unstable			Sustained Fault	
	t_c s	Critical Machines #	$V_{critical}$ (pu)	Unstable Machines #	$V^a_{critical}$ (pu)
			WSCC System		
Gen. #4	0.159	4	V(4) = 0.6496	4	V(4) = 0.6420
			Reduced Iowa System		
Raun	0.1924	5,6	V(5,6) = 18.999	5,6	V(5,6) = 18.4312
C.B. #3	0.204	12	V(12) = 11.808	2,5,6,10, 12,16,17	V(12) = 11.5305
Ft. Calhoun	0.354	16	V(16) = 12.788	2,5,6,10, 12,16,17	V(16) = 12.2501
Cooper	0.211	2	V(2) = 11.103	2,5,6,10, 12,16,17	V(2) = 11.0437
			IEEE System		
Gen. #2	0.190	2	V(2) = 5.026	2	V(2) = 4.9565
Gen. #3	0.480	2	V(2) = 5.305	2,3	V(2) = 5.2510
Gen. #4	0.340	4,5	V(4,5) = 7.686	4,5	V(4,5) = 7.5932
Gen. #5	0.400	4,5	V(4,5) = 11.886	4,5	V(4,5) = 11.720
Gen. #9	0.480	9	V(9) = 4.603	9	V(9) = 4.5988
Gen. #13	0.340	8,9	V(8,9) = 1.559	1,2,3,4,5, 6,7,8,9,10, 11,12,13,14, 20	V(8,9) = 1.5532
Gen. #18	0.360	18	V(18) = 4.474	17,18	V(18) = 4.4710

[a]Maximum potential energy (along the sustained fault trajectory) for the group of machines that first becomes unstable.

Iowa system cause all seven generators along the Missouri River to lose synchronism. In the critically unstable condition, however, only one machine becomes unstable. The significance of this can be seen in the fact that the maximum potential energy for the group of seven generators is greater than 30 pu in the three cases. Yet the portion of that energy associated with the critical machine is fairly constant within 3-4 % of all cases.

Similar results are obtained with the IEEE system. $V_{critical}$ for the machines that first become unstable is fairly constant, between the critically unstable and sustained fault conditions, even when the mdoe of instability is changed by the sustained fault, e.g., faults at Generators #13 and #18.

Stability Assessment by Individual Machine Energy

For a given fault location, the total energy (i.e. kinetic and potential energy) at fault clearing is compared with $V_{critical}$, for the critical machines individually and as a group, when the system is critically stable and when it is critically unstable. The data are displayed in Table 5.2. In that table, the values of $V_{critical}$ are obtained from the sustained fault run. In addition, in the computation of V_{total} at fault clearing, the kinetic energy is calculated using equation (2.14) to give the correct energy separating the critical machines from the rest of the system. The critical clearing time, based on the value of V_{total} equal to $V_{critical}$, is also shown in Table 5.2.

Table 5.2. Stability assessment using individual machine energy

Fault Location	Critical Machines #	Critically Stable Case		Critically Unstable Case		Assessment Based on V_{cr}	
		t_c s	V_{total} pu	t_c s	V_{total} pu	$V_{critical}$ pu	Critical Clearing time-s
WSCC System							
Gen. #4	4	0.156	V(4) = 0.6316	0.159	V(4) = 0.6575	V(4) = 0.6420	0.1572
Reduced Iowa System							
Raun	5,6	0.192	V(5) = 1.6227	0.1924	V(5) = 1.8248	V(5) = 1.6233	0.1920
			V(6) = 18.0393		V(6) = 18.3828	V(6) = 18.3309	0.1923
			V(5,6)= 17.2026		V(5,6)= 18.9997	V(5,6)= 18.4312	0.1923
C.B. #3	12	0.200	V(12) = 11.0809	0.204	V(12) = 11.8321	V(12) = 11.5305	0.202
Ft. Calhoun	16	0.345	V(16) = 11.8768	0.356	V(16) = 12.7942	V(16) = 12.2501	0.350
Cooper	2	0.204	V(2) = 9.9535	0.212	V(2) = 11.2988	V(2) = 11.0437	0.210
IEEE System							
Gen. #2	2	0.180	V(2) = 4.4653	0.190	V(2) = 5.2415	V(2) = 4.9565	0.186
Gen. #4	4,5	0.320	V(4) = 8.2416	0.340	V(4) = 9.8113	V(4) = 9.7834	0.340
			V(5) = 1.9624		V(5) = 2.1975	V(5) = 2.0284	0.3261
			V(4,5)= 6.4312		V(4,5)= 7.6083	V(4,5)= 7.5932	0.339
Gen. #5	4,5	0.380	V(4) = 1.3777	0.400	V(4) = 1.6322	V(4) = 1.4292	0.384
			V(5) = 12.1356		V(5) = 13.7196	V(5) = 13.3751	0.396
			V(4,5)= 10.3421		V(4,5)= 11.8870	V(4,5)= 11.720	0.398
Gen. #9	9	0.460	V(9) = 3.4310	0.480	V(9) = 4.6393	V(9) = 4.5988	0.479
Gen. #18	18	0.340	V(18) = 4.0335	0.360	V(18) = 4.5320	V(18) = 4.4710	0.358

Examining the data in Table 5.2, it can be seen that in every case the critical clearing time based on $V_{total/t_c} = V_{critical}$ checks well with the data obtained by time simulation, i.e., $t_{critical}$ falls between the critically stable and critically unstable clearing times. Furthermore, the transient stability assessment yields the correct prediction of stability (or instability) whether the cricical machines are checked individually or as a group. As seen from the data in Table 5.2, for all practical purposes the predicted critical clearing time is the same; e.g., for the Raun fault the critical t_c's based on V(5) or V(6) or V(5,6) are essentially the same.

Special cases

Three disturbances are of particular interest since the machines initially losing synchronism are different from the machines at which the fault is applied; the latter machines maintain synchronism in the critically unstable conditions. These cases, presented separately in Table 5.3 are:

- The Reduced Iowa System, where the initial conditions are altered so that 200MW of generation are shifted from Gen. #4 (Wilmarth) to Gen. #6 (Raun). With the fault applied at the Ft. Calhoun terminal (Gen. #16), the Raun generators (#5,6) first lose synchronism, while Gen. #16 does not.

- The IEEE system, with the same initial conditions. A fault at the terminal of Gen. #3 causes Gen. #2 to lose synchronism first;

Table 5.3. Stability assessment using individual machine energy: special cases

Fault Location	Critical Machines #	Critically Stable Case t_c s	Critically Stable Case V_{total} pu	Critically Unstable Case t_c s	Critically Unstable Case V_{total} pu	Assessment Based on V_{cri}: $V_{critical}$ pu	Assessment Based on V_{cri}: Critical Clearing time-s
Reduced Iowa System							
Ft. Calhoun (Gen. #16)	5,6	0.310	V(5) = 0.8884	0.3314	V(5) = 1.1105	V(5) = 0.9972	0.321
			V(6) = 6.6099		V(6) = 7.1924	V(6) = 7.0145	0.325
			V(5,6) = 6.5213		V(5,6) = 7.0131	V(5,6) = 6.9214	0.327
			V(16) = 6.5695		V(16) = 9.4860	V(16) = 11.7178	
IEEE System							
Gen. #3	2	0.460	V(2) = 3.7016	0.480	V(2) = 5.3691	V(2) = 5.2510	0.479
			V(3) = 7.2902		V(3) = 7.6281	V(3) = 9.2627	
Gen. #13	8,9	0.320	V(8) = 0.7313	0.340	V(8) = 0.8135	V(8) = 0.8021	0.336
			V(9) = 0.6746		V(9) = 0.7544	V(9) = 0.7470	0.338
			V(8,9) = 1.3621		V(8,9) = 1.5599	V(8,9) = 1.5532	0.339
			V(13) = 12.9372		V(13) = 15.5678	V(13) = 17.0606	

and with a fault at the terminal of Gen. #13, the first generators to become unstable are Gen. #8,9. Again the data in Table 5.3 show that with the use of $V_{critical}$ for individual machines the correct mode of instability is determined, and the critical clearing time for the critical machines can be predicted with the same accuracy obtained by time simulation.

Correspondence with the Controlling U.E.P. Concept

In reference (41) the controlling u.e.p. concept was validated; i.e., for the trajectory of the disturbed multi-machine power system, there is an u.e.p. that determines stability. It was also reported that in the critically stable (or unstable) trajectory, the critical machines pass at or near their u.e.p. values, and that for all practical purposes the system critical transient energy (with appropriate kinetic energy corrections) is equal to the system potential energy at the u.e.p. This section explores the correspondence, if any, between the controlling u.e.p. concept and the critical transient energy of the individual machines.

It maybe recalled that the energy of individual machines is obtained by time simulation along the actual system trajectory. Since the trajectory may not actually pass through the u.e.p., a procedure is adopted which assumes that the critically unstable trajectory crosses the Principal Energy Boundary Surface (PEBS) near the controlling u.e.p. (The PEBS is defined by the following constructive procedure. Starting from the post-fault s.e.p., go out in every direction in angle-space. Along each ray emanating from the s.e.p., find the first point where the potential energy function becomes a maximum. The set of points $\underline{\theta}$ found in this way characterizes the PEBS of interest. See references (38, 39).)

The procedure involves the following steps:

1. The controlling u.e.p. is determined by the method discussed in reference (41). A Davidon-Fletcher-Powell minimization technique is used to determine the u.e.p. given the post-fault system condition and initial estimate.

2. The critically cleared but unstable trajectory is obtained. For that run, the individual machine energies are computed, and the instant of crossing the PEBS (V_{PE} is maximum) is determined.

3. An instant of two time steps (about 0.08s), before the PEBS is crossed on the critically unstable trajectory, is selected. Starting from that instant, the potential energy of the individual machines is recomputed using the values of machine angles at the controlling u.e.p. as the upper limit of integration, in the energy calculation. It is assumed that the transient energies of individual machines thus obtained would be essentially the same as with the trajectory actually passing through the u.e.p.

Table 5.4. Individual machine energies at UEP, pu

Energy	Fault at Gen. # 2	Fault at Gen. # 3
Critical generator for controlling u.e.p.	2	2,3
$V_u = V_{PE}$ at u.e.p. (using Ref. (41)).	3.4686	10.1806
$V_{PE}(1)$	0.5367	-0.7324
$V_{PE}(2)$	4.7632	5.0710
$V_{PE}(3)$	-0.0472	9.9939
$V_{PE}(4)$	-0.1139	0.2327
$V_{PE}(5)$	-0.1593	0.5453
$V_{PE}(6)$	-0.0630	-0.0139
$V_{PE}(7)$	-0.0732	-0.0371
$V_{PE}(8)$	-0.0891	-0.0431
$V_{PE}(9)$	-0.1032	-0.0793
$V_{PE}(10)$	-0.0892	-0.2731
$V_{PE}(11)$	-0.0753	-0.1931
$V_{PE}(12)$	-0.0790	-0.2761
$V_{PE}(13)$	-0.1039	-0.4213
$V_{PE}(14)$	-0.1193	-0.4421
$V_{PE}(15)$	-0.2003	-0.6392
$V_{PE}(16)$	-0.1632	-0.7921
$V_{PE}(17)$	-0.2249	-0.9963
$V_{PE}(18)$	-0.0413	-0.3263
$V_{PE}(19)$	-0.1239	-0.4632
$V_{PE}(20)$	-0.1132	-0.5713
$V_{PE}(\text{total}) = \Sigma V_{PE}(i)$	3.3175	9.5435

With the procedure outlined above, two cases in the IEEE system were investigated:

1. Fault at the terminal of Gen. #2, with only Gen. #2 losing synchronism in the critically unstable case. The controlling u.e.p. is that with Gen. #2 as the critical machine.

2. Fault at the terminal of Gen. #3, again with only Gen. #2 losing synchronism in the critically unstable case. However, the controlling u.e.p. is that with Gen. #2 and Gen. #3 as the critical machines.

The individual machine energies at the u.e.p. as computed by this procedure contain potential energy as well as kinetic energy components. The values of potential energy of the individual machines at the u.e.p. are given in Table 5.4 together with the total potential energy at the u.e.p. (V_u) computed by the method of reference (41), for the two above-mentioned disturbances.

Examining the data in Table 5.4, it can be noted that the system potential energy at the u.e.p. (as computed by the method of reference (41)) agrees fairly well with the sum of the potential energy components of the individual machine energies computed by the procedure described. The difference can be easily attributed to the variation in the procedures used. Thus, it can be concluded that the critical transient energy for the system as a whole is made up of contributions due to the individual machines. A question now arises as to the meaning of the components attributed to the critical machines. For these machines, the values of $V_{critical}$ (as given in Table 5.1) are compared with the values shown in Table 5.4. The comparison is shown in Table 5.5.

Table 5.5. Comparison of $V_{critical}$

Fault Location	V(2)		V(3)	
	V_{cr}	$V_{PE}^{(uep)}$	V_{cr}	$V_{PE}^{(uep)}$
Fault at Gen. #2	4.9568	4.7632		
Fault at Gen. #3	5.2510	5.0710	9.9939	9.2627

The data in Table 5.5 indicate that, for a given mode of instability, $V_{cr}(i)$ for each machine is the same as that machine's contribution to the system's potential energy at the controlling u.e.p. for the critical trajectory.

A point of considerable significance has been revealed by the present investigation, namely that for the controlling u.e.p. the large angles are associated with the generators that are severely disturbed even if some of them do not actually become unstable in the critically unstable case. This phenomenon has been observed, and briefly discussed, in the detailed investigations conducted on the Iowa system (43).

The data in Table 5.6 illustrate this point for the same two cases discussed above. In both cases, only Gen. #2 loses synchronism in the critically unstable case. The angle θ_2 is close to θ_2^u in both cases. The angle, θ_3, however, is close to its u.e.p. value only for the fault at Gen. #3, the value of θ_3 is far from that for u.e.p. based on Gen. #2 alone. The values of the energy as well as the rotor angles indicate that the controlling u.e.p. for that fault is based on both Gen. #2 and Gen. #3 together.

Table 5.6. Comparison of angle θ

	θ_2			θ_3		
	Critically Stable Run	Critically Unstable Run	u.e.p. Value	Critically Stable Run	Critically Unstable Run	u.e.p. Value
Fault at Gen. #2	116.23°	135.74°	125.14°	18.76°	-1.01°	-7.83°
Fault at Gen. #3	100.21°	118.32°	115.48°	99.44°	119.41°	178.81°

CHAPTER VI. MATHEMATICAL ANALYSIS OF INDIVIDUAL MACHINE ENERGY

The Concept of Partial Stability

In this chapter, a theoretical explanation of the individual machine energy is provided using the concept of partial stability. The formulation, definitions and theorems have been obtained from references (47, 48).

A brief explanation of the notations used in this chapter are now presented. Let V and W be arbitrary sets. Then V x W denotes the cross product of V and W. $V \subset W$ denotes that V is a subset of W. $x \in W$ denotes that x is an element of W. The notation $r: V \rightarrow W$ denotes a function or mapping r of V into W. R^n is a n-dimensional real space with a norm $||.||$ defined on it. Let R denote the real numbers, then $R^+ = [0,\infty)$.

Let $n>0$ and $m>0$ be two integers, and consider two continuous functions

$$\underline{f} : \Omega \times R^m \rightarrow R^n$$

$$\underline{g} : \Omega \times R^m \rightarrow R^m$$

Ω is a domain of R^n containing the origin. We assume that $\underline{f}(0,0) = 0$ and $\underline{g}(0,0) = 0$ and further that $\underline{f}$ and $\underline{g}$ are smooth enough in order that through every point of $\Omega \times R^m$ there passes one and only one solution of the differential equations.

$$\left.\begin{array}{l}\dot{\underline{x}} = \underline{f}(\underline{x}, \underline{y}) \\ \dot{\underline{y}} = \underline{g}(\underline{x}, \underline{y})\end{array}\right\} \dot{\underline{z}} = \underline{h}(\underline{z}) \qquad (6.1)$$

To shorten notation write $\underline{z}$ for the vector $(\underline{x}, \underline{y}) \varepsilon R^{n+m}$ and also $\underline{z}(t;t_0, \underline{z}_0) = (\underline{x}(t;t_0, \underline{z}_0), \underline{y}(t;t_0, \underline{z}_0))$ for solution of equation (6.1) starting from $\underline{z}_0$ at t_0.

Definitions

Stability The solution $\underline{z} = 0$ of (6.1) is stable with respect to $\underline{x}$ if for any t_0 and any A, $0<A<H$ there is a $\lambda(A, t_0)>0$ such that for $t \geq t_0$ and $||\underline{z}_0|| \leq \lambda$ imply that $||\underline{x}(t;t_0, \underline{z}_0)||<A$.

Uniform stability The solution $\underline{z} = 0$ of (6.1) is uniformly stable with respect to $\underline{x}$ if for any A, $0<A<H$, there is a $\lambda(A)>0$ such that for $t \geq t_0$ and $||\underline{z}_0||\leq\lambda$ imply that $||\underline{x}(t;t_0, \underline{z}_0)||<A$.

Asymptotic stability The solution $\underline{z} = 0$ of (6.1) is asymptoticallly stable with respect to $\underline{x}$ if it is stable with respect to $\underline{x}$ and if for any t_0 there is a $\lambda_1(t_0) > 0$ such that $||\underline{z}_0|| \leq \lambda_1$ imply that $||\underline{x}(t;t_0, \underline{z}_0)|| \to 0$ as $t \to \infty$.

A function ψ is said to be a C^1 function if it is continuously differentiable.

A continuous function $\psi: R^+ \to R$ is said to belong to class K, $\psi \varepsilon K$ if

i) $\psi(0) = 0$

ii) $\psi(r_1) > \psi(r_2)$ whenever $r_1 > r_2$.

Stability theorems

Theorem 1 If there exists a C^1 function $V:\Omega x R^m \rightarrow R$ such that for some $a \varepsilon K$ and every $(\underline{x}, \underline{y}) \varepsilon \Omega \times R^m$:

i) $V(\underline{x}, \underline{y}) \geq a(||\underline{x}||)$, $V(0, 0) = 0$

ii) $\dot{V}(\underline{x}, \underline{y}) \leq 0$

then the origin $\underline{z} = 0$ is stable with respect to $\underline{x}$

Moreover, if for some $b \varepsilon K$ and every $(\underline{x}, \underline{y}) \varepsilon \Omega \times R^m$:

iii) $V(\underline{x}, \underline{y}) \leq b(||\underline{x}|| + ||\underline{y}||)$

then the origin $\underline{z} = 0$ is uniformly stable with respect to $\underline{x}$.

Theorem 2 Suppose there exists a C^1 function $V: \Omega \times R^m \rightarrow R$ such that for some functions $a, c \varepsilon K$ and every $(\underline{x}, \underline{y}) \varepsilon \Omega \times R^m$:

i) $V(\underline{x}, \underline{y}) \geq a(||x||)$, $V(0, 0) = 0$

ii) $\dot{V}(\underline{x}, \underline{y}) \leq - c(V(\underline{x}, \underline{y}))$

then the origin $\underline{z} = 0$ is asymptotically stable with respect to $\underline{x}$.

Theorem 3 Suppose there exists a C^1 function $V: \Omega \times R^m \rightarrow R$ such for some $\emptyset \varepsilon K$ and every $(\underline{x}, \underline{y}) \varepsilon \Omega \times R^m$:

i) $V(\underline{x}, \underline{y}) \geq \emptyset(||\underline{x}||)$, $V(0,0) = 0$

ii) $\dot{V}(\underline{x}, \underline{y}) \leq 0$ on $\Omega \times R^m$

Let $S = \{(\underline{x}, \underline{y}) \varepsilon \Omega \times R^m: \dot{V}(\underline{x}, \underline{y}) = 0\}$. If S does not contain a whole trajectory of $\dot{\underline{z}} = \underline{h}(\underline{z})$, other than the origin $\underline{z} = 0$, and if all solutions of (6.1) determined by $||\underline{z}_0||<\lambda$ are bounded in $\underline{y}$, then the equilibrium $\underline{z} = 0$ is asmptotically stable with respect to $\underline{x}$.

Application to the Power System Problem

Repeating equation (2.5) for the sake of convenience, we see that the dynamics of a power system are governed by a system of differential equations

$$M_i \dot{\tilde{\omega}}_i = P_i - P_{ei} - \frac{M_i}{M_T} P_{COI}$$
$$\dot{\theta}_i = \tilde{\omega}_i \qquad i = 1, 2, 3, ---, n \tag{6.2}$$

Another fact of considerable significance, which has been discussed in the previous chapters is that when a power system is disturbed, either one machine or a group of machines loses synchronism with the rest of the system. This machine or group of machines is commonly known as the critical group. The rest of the system which remains stable forms the non-critical group. Thus, the power system stability problem can be cast as a partial stability problem, where the stability of the entire system is studied with respect to the critical group of machines.

Without loss of generality, let machine 1 be the critical machine. Then referring to equation (6.1) it is seen that:

$$\underline{x} = \begin{bmatrix} \tilde{\omega}_1 \\ \theta_1 \end{bmatrix}$$

$$\underline{y} = \begin{bmatrix} \underline{\tilde{\omega}_i} \\ \underline{\theta_i} \end{bmatrix} \quad i = 2, 3, ---, n$$

$$\underline{z} = \begin{bmatrix} \underline{\tilde{\omega}_i} \\ \underline{\theta_i} \end{bmatrix} \quad i = 1, 2, ---, n$$

Thus, the power system equations can be cast in the same form as equation (6.1) and the various definitions of partial stability can be applied.

In order to apply the various theorems of stability, consider the individual machine energy function developed in Chapter II as a candidate V-function. Repeating equation (2.7)

$$V_i = \frac{1}{2} M_i \tilde{\omega}_i^2 - P_i(\theta_i - \theta_i^s) + \sum_{\substack{j=1 \\ j \neq i}}^{n} C_{ij} \int_{\theta_i^s}^{\theta_i} \sin\theta_{ij} d\theta_i$$

$$+ \sum_{\substack{j=1 \\ j \neq i}}^{n} D_{ij} \int_{\theta_i^s}^{\theta_i} \cos\theta_{ij} d\theta_i + \frac{M_i}{M_T} \int_{\theta_i^s}^{\theta_i} P_{COI} \, d\theta_i \tag{6.3}$$

Equation (6.3) can also be written as

$$V_i = V_{KEi} + V_{PEi} \tag{6.4}$$

where V_{KEi} = kinetic energy of machine i

V_{PEi} = potential energy of machine i.

Heuristic justification for the positive definiteness of V_i will be used. Consider the critical machine (or critical group) after the system had been subjected to a disturbance, and upon removal of the disturbance, e.g. at and after fault clearing. As the system moves along the post-disturbance trajectory the kinetic energy is being converted into potential energy as pointed out in Chapter III. Stability is determined by the network's ability to convert all the kinetic energy that tends to separate the critical machines from the rest of the system into potential energy. In that period, the total transient energy is constant and is made up of kinetic energy and potential energy. Both components of energy are positive. This is illustrated for the one-machine-infinite bus system in Figure 2.1.

Again let machine 1 be the critical machine. From the discussion in Chapter III, $V_{1/critical}$ is reached when $V_{1/critical} = V_{PE1/max}$ and $V_{PE1/max} > 0$. Thus, in all theorems the conditions that $V(\underline{x}, \underline{y})$ be positive definite is satisfied. Also,

$$\dot{V}_i = [\, M_i \dot{\tilde{\omega}}_i - P_i + P_{ei} + \frac{M_i}{M_T} P_{COI} \,] \tilde{\omega}_i \tag{6.5}$$

or

$$\dot{V}_i = \dot{V}_{KEi} + \dot{V}_{PEi} \tag{6.6}$$

After the fault is cleared, the critical machine 1 satisfies the equation

$$M_1 \dot{\tilde{\omega}}_1 = P_1 - P_{e1} - \frac{M_1}{M_T} P_{COI} \qquad (6.7)$$

Therefore,

$$\dot{V}_{KE1} = - \dot{V}_{PE1}$$

or

$$\dot{V}_1 = 0 \qquad (6.8)$$

Hence, by its inherent nature, the time derivative of the energy function is identically zero and therefore can never be negative definite but only negative semi-definite.

All the conditions of Theorem 1 are thus satisfied and partial stability of the power system with respect to the critical machine is assured. However, condition (ii) of Theorem 2 is not satisfied, as a result of which Theorem 2 cannot be applied to determine partial asymptotic stability.

To arrive at a condition of partial asymptotic stability, Theorem 3 is applied. This theorem makes use of the concept of invariant sets. On closely examining Theorem 3, it can be seen that the individual machine energy satisfies conditions (i) and (ii). Now consider the invariant set $S = \{(\underline{x}, \underline{y}) \varepsilon \Omega \times R^m : \dot{V}(\underline{x}, \underline{y}) = 0\}$. As seen in Chapter II, the entire stability analysis is done for the post-fault system with the post-fault stable equilibrium point as the origin. Also, the individual machine energy is calculated with respect to the post-fault stable equilibrium point. The set S consists of all those points in state space, from the instant of clearing to the instant the potential energy of the critical group of machines peaks. A whole trajectory corresponds to the right hand side of equation (6.1) being zero. In a power system, the post-fault trajectory satisfies this condition only at the s.e.p. or at the u.e.p. The former is the origin of the system; the latter is at the boundary of the region of stability. Thus, if the post-fault trajectory is asymptotically stable $V(\underline{x}, \underline{y}) < V_{critical}$, and the trajectory is constrained to the region of stability. Thus, for asymptotic stability the trajectory terminates at the s.e.p. For this condition $\dot{\underline{z}} = \underline{h}(\underline{z}) = 0$, and the invariant set S contains no whole trajectory other than the origin $\underline{z} = 0$. Also, we are assured that the solutions of (6.1) determined by $||\underline{z}_0|| < \lambda$ are bounded in $\underline{y}$, because this corresponds to the non-critical group which remain in synchronism, i.e., partially stable. Hence, Theorem 3 guarantees partial asymptotic stability with respect to the critical group.

CHAPTER VII. CONCLUSION

This dissertation used the critical energy of individual machines to assess the transient stability of a multi-machine power system. The stability assessment using individual machine energy was confirmed in terms of critical clearing times obtained by time simulation of the system differential equations.

The dissertation sugggests that a realistic assessment of transient stability requires examining the behaviour of a critical group of machines in the post-disturbance period. By comparing their transient energy at clearing with their maximum potential energy along the post-disturbance trajectory, transient stability assessment can be made. This is in contrast to the prevailing practice, in the past two decades, of assessing transient stability via a system-wide energy function.

Detailed analysis of the transient energy and its components along the trajectory has pointed the need for making corrections to the energy values based on system-wide functions (43). The present work, and the detailed supporting data obtained by simulation studies, have shown that transient stability of a multi-machine power system can be fully explained by energy functions for individual machines. The apparent contradiction between the data obtained by simulation (and supported further by heuristic argument and/or physical behavior) and the prevailing theory has been fully resolved by the theoretical work developed in Chapter VI. The concept of partial stability satisfactorily reconciles the theory and the data obtained. This is the first time that a theory proposed yields correct results and matches the power system behaviour as evidenced by the numerous simulation studies performed.

The following conclusions can be drawn from the data presented in Chapter V.

- The energy function for an individual machine, or for a group of machines (equations (2.7) and (2.16)) gives its correct transient energy on the post disturbance trajectory.

- The potential energy maximum in the expression for the individual machine energy is indicative of the network's transient energy-absorbing capacity for that machine, i.e., its capacity to convert transient kinetic energy (tending to separate the machine from the rest of the system) into potential energy. This potential energy maximum is $V_{critical}$ for that machine for a given disturbance location. Similar conclusions can be stated for a group of machines tending to separate from the system.

- $V_{critical}$ for an individual machine (or a group of machines) for a given disturbance location is fairly constant for critically unstable conditions or with more severe disturbance. Values of $V_{critical}$ obtained from sustained fault runs are slightly conservative (by about 2-3 %) as seen in Table 5.1.

- In a multi-machine power system, a machine or a group of machines separate from the rest of the system when the associated network energy absorbing capacity is exceeded by the transient energy it has at the end of the disturbance. For these conditions, the total transient energy possessed by these machines at the instant the disturbance is removed exceeds their corresponding values of $V_{critical}$. This fact is of great significance in explaining instability in a multi-machine power system.

- Assessing first swing transient stability by the individual machine energy accurately predicts:
  - (i) the mode of instability, even in very complex situations (see special cases in Table 5.3), and
  - (ii) critical clearing time (results are for all practical purposes <u>identical</u> to those obtained by time solutions).

- By making a number of sustained fault runs at desired network locations, the needed information for direct transient stability assessment is obtained.

- Investigations conducted on the correspondence between the individual machine energies indicate that:
  - (i) $V_{i/critical}$ corresponds to the contribution of machine i to the system potential energy V_u at the controlling u.e.p.,
  - (ii) the validity of the controlling u.e.p. concept is upheld; when $V_{i/critical}$ is just exceeded, the system trajectory will reach the PEBS at a point not far from the u.e.p., and
  - (iii) a point of considerable significance is the confirmation that the controlling u.e.p. is determined by severely

disturbed generators even if some of them do not actually lose synchronism in the critically unstable case.

- A theoretical explanation for using individual machine energy can be provided using the concept of partial stability.

Finally, the results obtained in the course of this investigation clearly show that instability in a multi-machine power system depends on where the transient energy resides in the system and how it is exchanged between each of the critical machines and the rest of the system. Thus, the individual machine behavior is the key factor in determining whether synchronism is maintained or lost.

Suggestions for Future Research

On the basis of the investigation carried out in this research endeavor, some of the problems that need further attention are:

- Defining a transient energy margin based on the individual machine energy.
- Proposing a transient security assessment scheme based on the the above mentioned energy margin.
- Establishing a decision criterion for the operator that can be used on a real time basis for operational decision-making.

- Including effects of more detailed power system model. This dissertation utilized a classical model. This model provides a fairly accurate understanding of first swing stability (instability). However, for practical assessment of transient stability it maybe necessary to use a more detailed model which includes the following features.

    - Unreduced network representation.

    - Non-linear load modeling.

    - Better modeling of "field effects."

REFERENCES

1. Crary, S. B. Power System Stability. Vol. I. - Vol. II. New York: John Wiley & Sons, Inc., 1948.

2. Kimbark, E. W. Power System Stability. Vol. I. Elements of Stability Calculations. New York: John Wiley & Sons, Inc., 1948.

3. Criteria of Stability of Electric Power Systems. A report. Union Institute of Scientific and Technological Information and the Academy of Sciences of the U.S.S.R. Electric Technology and Electric Power Series, Moscow, 1971.

4. Magnusson, P. C. "Transient Energy Method of Calculating Stability." AIEE Trans. PAS-66 (1947): 747-755.

5. Aylett, P. D. "The Energy Integral Criterion of Transient Stability Limits of Power Systems." Proceedings IEEE 105(C) (1958): 527-536.

6. Gless, G. E. "Direct Method of Lyapunov Applied to Transient Power System Stability." IEEE Trans. PAS-85 (February 1966): 164-179.

7. El-Abiad, A. H. and Nagappan, K. "Transient Stability Regions of Multimachine Power Systems." IEEE Trans. PAS-85 (February 1966): 158-168.

8. Pai, M. A., Mohan, M. A. and Rao, J. G. "Power System Transient Stability Regions Using Popov's Method." IEEE Trans. PAS-89 (May/June 1970): 788-794.

9. Yu, Y. N. and Vongasuriya, K. "Nonlinear Power System Stability Study by Lyapunov Function and Zubov's Method." IEEE Trans. PAS-86 (December 1967): 1480-1485.

10. Willems, J. L. "Improved Lyapunov Function For Transient Power-System Stability." Proceedings IEE 115 (September 1968): 1315-1317.

11. Luders, G. A. "Transient Stability of Multimachine Power Systems Via the Direct Method of Lyapunov." IEEE Trans. PAS-90 (January/February 1971): 23-36.

12. Carton, E. and Ribbens-Pavella, M. "Lyapunov Methods Applied to Multi-Machine Transient Stability with Variable Inertia Coefficients." Proceedings IEE 118 (November 1971): 1601-1606.

13. Uyemura, K. and Matsuki, J. "Computational Algorithm for Evaluating Unstable Equilibrium States in Power Systems." Electrical Engineering in Japan 92 (July/August 1972): 41-47.

14. Mansour, M. "Generalized Liapunov Function for Power Systems." IEEE Trans. AC-19 (June 1974): 247-248.

15. Pai, M. A. and Murthy, P. G. "New Liapunov Functions for Power Systems Based on Minimal Realizations." International Journal of Control 19 (February 1974): 401-415.

16. Pai, M. A. and Varwandkar, S. D. "On the Inclusion of Transfer Conductances in Lyapunov Functions for Multimachine Power System." IEEE Trans. AC-22 (December 1977): 983-985.

17. Ribbens-Pavella, M. "Transient Stability of Multimachine Power Systems by Lyapunov's Direct Method." Paper 71 CP17PWR IEEE PES Winter Meeting, New York, 1971.

18. Kitamura, S., Dohnomoto, T. and Kurematsu, Y. "Construction of a Lyapunov Function by the Perturbation Method and its Application to the Transient Stability Problem of Power Systems with Non-Negligible Transfer Conductances." Int. J. of Control 26 (1977): 405-420.

19. Lugtu, R. L. and Fouad, A. A. "Transient Stability Analysis of Power Systems Using Lyapunov's Second Method." IEEE Conference Paper No. C72145-6.

20. Tavora, C. J. and Smith, O. J. M. "Characterization of Equilibrium and Stability in Power Systems." IEEE Trans. PAS-91 (May/June 1972): 1127-1130.

21. Prabhakara, F. S. and El-Abiad, A. H. "A Simplified Determination of Transient Stability Regions for Lyapunov Methods." IEEE Trans. PAS-94 (March/April 1975): 672-689.

22. Gupta, C. L. and El-Abiad, A. H. "Determination of the Closest Unstable Equilibrium State for Lyapunov Methods in Transient Stability Studies." IEEE Trans. PAS-94 (September/October 1976): 1699-1712.

23. Ribbens-Pavella, M., Grujic, Lj. T. and Sabatel, J. "Direct Methods for Stability Analysis of Large-Scale Power Systems." Proc. of the IFAC Symposium, New-Delhi, India, August 1979.

24. Fouad, A. A. "Stability Theory-Criteria for Transient Stability." Proc. Conference on Systems Engineering for Power: Status and Prospects, Henniker, N.H., 1975.

25. Ribbens-Pavella, M. "Transient Stability of Multimachine Power Systems by Lyapunov's Direct Method." Proc. of Seminar on Stability of Large Scale Power Systems at the University of Liege, Liege, Belgium, 1972.

26. Bellman, R. "Vector Lyapunov Functions." SIAM Journal 1 (1962): 32-34.

27. Bailey, F. N. "The Application of Lyapunov's Second Method to Interconnected Systems." SIAM Journal 3 (1966): 443-462.

28. Pai, M. A. and Narayana, C. L. "Stability of Large Scale Power Systems." Proc. 6th IFAC Congress 6(1975): 1-10.

29. Michel, A. N. "Stability Analysis of Interconnected Systems." SIAM Journal 12 (August 1974): 554-579.

30. Michel, A. N. "Stability Analysis and Trajectory Behaviour of Composite Systems." IEEE Trans. CS-22 (April 1975): 305-312.

31. Michel, A. N. and Miller, R. K. Qualitative Analysis of Large-Scale Dynamical Systems. New York: Academic Press, 1977.

32. Weissenberger, S. "Stability Regions of Large Scale Systems." Automatica 9 (1973): 309-321.

33. Araki, M. "Application of M-matrices to the Stability Problems of Composite Dynamical Systems." Journal of Mathematical Analysis and Applications 52 (1975): 309-321.

34. Jocic, Lj. B., Ribbens-Pavella, M. and Siljak, D. D. "Multimachine Power Systems: Stability, Decomposition and Aggregation." IEEE Trans. AC-23 (April 1978): 325-332.

35. Chen, Y. K. and Schinzinger, R. "Lyapunov Stability of Multi-Machine Power Systems Using Decomposition-Aggregation Methods." Paper-A-80-03604, IEEE PES Winter Meeting, New York, 1980.

36. Pai, M. A. and Vittal, V. "Multimachine Power System Stability Using Vector Lyapunov Function with Inertial Center Decomposition." Proceedings of the Midwest Power Symposium, Urbana, Illinois, October 1981.

37. Athay, T., Podmore, R. and Virmani, S. "A Practical Method for Direct Analysis of Transient Stability." IEEE Trans. PAS-98 (1979): 573-584.

38. Athay, T., Sherket, V. R., Podmore, R., Virmani, S. and Puech, C. "Transient Energy Analysis." Proc. Conference on Systems Engineering for Power: Emergency Operating State Control, Davos, Switzerland, 1979.

39. Kakimoto, N., Ohsawa, Y. and Hayashi, M. "Transient Stability Analysis of Electric Power Systems via Lure' Type Lyapunov Function." Proceedings of IEE (Japan). 98 (May/June 1978): 63-78.

40. Bergen, A. R., and Hill, D. J. "A Structure Preserving Model for Power System Stability Analysis." IEEE Trans. PAS-100 (March 1981): 25-35.

41. Fouad, A. A. and Stanton, S. E. "Transient Stability Analysis of a Multi-Machine Power System. Part I: Investigation of System Trajectory; and Part II: Critical Transient Energy." IEEE Trans. PAS-100 (August 1981): 3408-3424.

42. Fouad, A. A., Stanton, S. E., Mamandur, K. R. C. and Kruempel, K. C. "Contingency Analysis Using the Transient Energy Margin Technique." Paper 81SM397-9. IEEE PES Summer Meeting, Portland, 1981.

43. Fouad, A. A., Kruempel, K. C., Mamandur, K. R. C., Pai, M. A., Stanton, S. E. and Vittal, V. "Transient Stability Margin as a Tool for Dynamic Security Assessment." Electrical Power Research Institute Report EL-1755, March 1981.

44. Athay, T. M. and Sun, D. I. "An Improved Energy Function for Transient Stability Analysis." Proceedings of the International Symposium on Circuits and Systems, April 1981.

45. Anderson, P. M. and Fouad, A. A. Power System Control and Stability. Vol. I. Ames, Iowa: The Iowa State University Press, 1977.

46. Bills, G. W. "On-line Stability Analysis Study." Final Report for Project RP90-1, Edison Electric Institute, October 1970.

47. Rouche, N., Habets, P. and Laloy, M. Stability Theory by Liapunov's Direct Method. New York: Springer-Verlag, 1977.

48. Rumyantsev, V. V. "On the Stability with Respect to a Part of the Variables." Symposia Math. New York: Academic Press, 1971.

ACKNOWLEDGMENTS

I would like to express my heartfelt gratitude to my major professor and mentor Dr. A. A. Fouad for the time and attention he has devoted to me and to this work. His sustained interest, encouragement and constant reminder to view the system from a physical aspect were of immense help in formulating and solving this research problem.

I would like to thank Dr. Anthony N. Michel, Dr. M. A. Pai and Dr. Richard K. Miller for their valuable time, encouragement and guidance provided. I would also like to thank Dr. Ken Kruempel for his help with the various computer programs. Special thanks are extended to Dr. R. Grover Brown for his participation as a member of my graduate committee.

I am also indebted to the Electrical Engineering Department and Engineering Research Institute at the Iowa State University for their financial assistance.

I would like to thank my parents and my wife, Sunanda, for their encouragement throughout my graduate program.

Thanks are also extended to Sheila Hilts for typing this thesis.

APPENDIX: COMPUTER PROGRAMS

Three computer programs were utilized in the research reported upon in this dissertation.

TESA

TSWING

INDENERGY

1. Transient-Energy Stability Analysis, (TESA), had been provided by Systems Control, Inc. (43). The TESA program estimates the unstable equilibrium angles, the critical energy, and the energy at the time of fault clearing. Though this program was not specifically used to generate the data in this dissertation, several subroutines were used in the program TSWING.

2. TSWING. This program was developed for the research conducted in reference (43). It simulates the disturbance by time simulation. At each time step it computes and plots various parameters. It includes the following basic features:

   - Generates the faulted and the post-fault Y-Buses.
   - Computes and plots rotor angles and velocities with respect to the system inertial center.
   - Computes and plots system kinetic energy and potential energy.
   - Injects additional disturbances, in the form of addition generation or load.

3. INDENERGY. This program was developed specifically for this dissertation work. It uses TSWING as the main body of the program, and at each time step branches into a subroutine to calculate individual machine energies. The potential energy calculation of each machine requires numerical integration and is done using trapezoidal rule as follows.

$$V_{P.E._i} = - \int_{\theta_i^s}^{\theta_i} \left(P_i - P_{ei} - \frac{M_i}{M_T} P_{COI}\right) d\theta_i$$

or

$$V_{P.E._i} = - \int_{\theta_i^s}^{\theta_i} f_i(\underline{\theta}) d\theta_i$$

$$V_{P.E._i}(k+1) = V_{P.E._i}(k) + [\ -f_i(\underline{\theta})(k+1) - f_i(\underline{\theta})(k)] \cdot \frac{1}{2} \cdot [\theta_i(k+1) - \theta_i(k)]$$